University of Mustansiriyah

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Under Graduate Study

Semester-I

Lectures on Quantum Mechanics

An Introduction

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Course Syllabus

Chapter One
Physical Foundations of Quantum Mechanics

1−1 Introduction
1−2 Quantum mechanics (Q.M)
1−3 The needs of Q.M
1-4 Phenomenon couldn't be interpreted by classical physics
1−4−1 Black body radiation
1−4−2 Atom stability
1−4−3 Photo electric effect
1−5 Natural duality of matter and radiation
1−6 Uncertainty principle
1−7 Correspondence principle

Chapter Two
Elementary Properties of Quantum Mechanics

2-1 Introduction
2-2 Wave function and its interpretation
2-3 Mathematical interpretation of the wave function
2-4 Derivation of Schrödinger wave equation
2-5 Operators
2-6 Operators properties
2-7 Operator equation
2-8 Eigen value equation
2-8-1 Eigen value and eigen functions
2-9 Observables
2-10 Expectation values
2-11 Variance
2-12 Equation of motion and constant of motion
2-13 Degeneracy
2-14 Conservation of probability and probability current density
2-15 Quantized states
2-16 Parity

Chapter Three
Simplified quantum systems

3-1 Introduction
3-2 Free particle

3-3 Schrödinger equation in one dimension
3-4 Potential step (finite and infinite height)
3-5 Particle in a box in one dimension
3-6 Particle in a box in three dimension
3-7 Density of states
3-8 Potential barrier penetration
3-9 Potential well

Chapter Four
Simple Harmonic Oscillator

4-1 Introduction
4-2 Hamiltonian Function
4-3 Solution of Schrödinger equation
4-4 Analytical treatment (energy eigen values and eigen wave functions)
4-5 Operator treatment (energy eigen values and eigen wave functions)
4-6 Comparison with classical theory

Chapter Five
One-electron atoms

5-1 Spherical symmetrical potential
5-2 Schrödinger equation
5-3 Solution of Schrödinger equation using the separation of variables
5-4 Wave functions (Azimuthally and zenithal parts, Spherical harmonics, Radial part)
5-5 Hydrogen atom
5-6 Wave function
5-7 Energy levels
5-8 Angular momentum Spin angular momentum
5-9 Pauli matrices

Chapter Six
Correction Methods

6-1 Introduction
6-2 Time Independent None Degenerate Perturbation theory
6-3 Applications
6-4 Time Independent Degenerate Perturbation theory
6-5 Applications
6-6 Variation method
6-7 Applications

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Chapter One
Physical Foundations of Quantum Mechanics

1-1 Introduction

In this chapter, we will attempt to identify the reasons for using quantum mechanics that has the credited for move up the development of most branches of modern physics.

1-2 Quantum mechanics

Quantum mechanics defined as the theoretical tool that is uses to investigate the microscopic systems such as atomic and nuclear systems. Therefore, this kind of mechanics deals with phenomena that arise from microscopic systems which invisible. However, it’s classical counterpart (classical mechanics) used to study and analyze large systems or macroscopic systems.

1-3 Necessity of Q.M.

That the necessity of quantum mechanics arises due to the failure of classical physics (Newton's mechanics and classical electromagnetic theory) in the interpretation of many physical phenomena such as black-body radiation, atom stability, photo-electric effect, Compton effect, pair production, matter annihilation, generating spectral x-ray lines.

1-4 Phenomenon could not interpreted by classical physics

1-4-1 Black body radiation

The best black body that could be imaginable is a cavity in any body which its walls at a certain temperature. The atoms forming the gap walls emit at the same time absorb electromagnetic radiation. When the amount of radiation that atoms emit reaches the same amount of radiation as it absorbs, the gap reaches the radiation equilibrium. Experiments have shown that in the case of equilibrium, the radiation has a specific energy distribution curve. Each frequency has a certain energy density that is weakly dependent on the temperature of the cavity’s wall and does not depend on the cavity material. The variation of the energy density as a function of the wavelength is shown in the figure below;

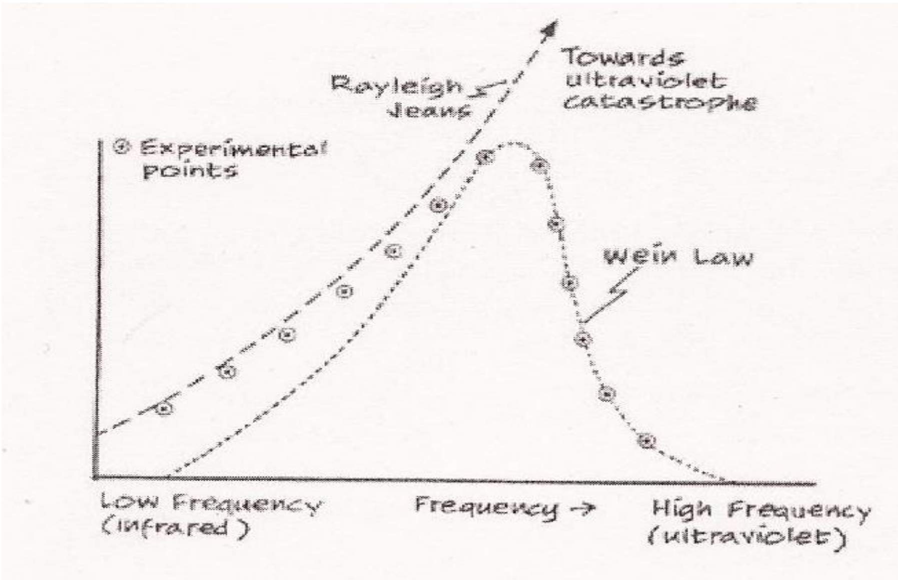

Figure (1-1): Energy density distribution of black body radiation.

Many scientists have tried to explain this curve using the ideas of classical physics, but failed to do so. Among the most of these attempts the following in below;

1- Wine's attempt

Wine introduced the following mathematical relationship to describe the above the curve of energy density distribution:

$$E(\lambda)d(\lambda) = \frac{C_1}{\lambda^5} e^{C_2/\lambda T} d\lambda \qquad \text{……………………..(1−1)}$$

Where C_1 and C_2 are constants, λ is the wavelength and T temperature. This relationship, which called the Wine's law of radiation, has failed to explain the curve of energy density distribution at low frequencies (long wavelength).

2- Rayleigh's attempt

Rayleigh introduced another attempt to describe blackbody radiation by deriving the following mathematical relationship to describe the distribution of energy density as a function of frequency;

$$E(\lambda)d(\lambda) = \frac{8\pi}{\lambda^4}\, kT\, d\lambda \qquad \text{.............................(1-2)}$$

Where K is the Boltzmann constant. Equation (1-2) is failed again to give significant interpretation of the curve in figure (1-1) at high frequencies (short wavelength). This relationship called the Rayleigh-Jeans law of radiation.

Thus, the radiation curve of the black body remained without convincing explanation until 1900, when Max Planck gave a precise interpretation of the curve using new ideas. Where he assume the following: -

1- The atoms of the gap walls behave like atomic oscillators, each vibrating at a certain frequency (υ).

2- Each oscillator absorbs or radiates a radiation proportional to its frequency. i.e.

$$E = h\upsilon \qquad \text{..............................(1-3)}$$

Where E is the energy emitted or absorbed by the atom and h is the constant of proportionality and is the same for all oscillators. In this way, we conclude that oscillators, when absorb or emits electromagnetic radiation, their energy increase or decrease by amount equal to ($h\upsilon$). Therefore, the relationship (1-3) implies that the energy of the oscillators being quantized. i.e.

$$E = \mathrm{n}h\upsilon \qquad \text{..............................(1-4)}$$

Using these ideas, Planck enable to express the density of the black body radiation energy by the following relation:

$$E(\upsilon)d\upsilon = \frac{8\pi h\upsilon^3}{C^3}\,\frac{d\upsilon}{e^{h\upsilon/KT}-1} \qquad \text{.............................(1-5)}$$

This relationship, which called the Planck radiation law, is conclusively agreed to the curve shown in figure (1-1) and for all frequencies when the constant value of h is equal to 6.6256 × 10-34 J.s, which was later called Planck constant.

Example: Express the energy of black body radiation, i.e. equation (1-5) in term of wavelength.

Solution:

Let,

$$E(\upsilon)\, d\upsilon = -E(\lambda)\, d\lambda$$

Since,

$$\upsilon = \frac{c}{\lambda}$$

Then,

$$d\upsilon = -\frac{c}{\lambda^2} d\lambda$$

Substitute in equation (1-5) one obtain:

$$E(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{e^{hc/\lambda KT} - 1}$$

Example: Show that, Planck radiation low reduced to Wien's radiation law for high frequencies and to Rayleigh – Jeans radiation law for very low frequencies.

Solution:

i) For high υ (i.e. for low λ):

$$e^{hc/\lambda KT} >> 1$$

So,

$$E(\lambda)d\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda KT} d\lambda$$

ii) For low υ (i.e. for high λ):

$$e^{hc/\lambda KT} - 1 \approx \frac{hc}{\lambda KT}$$

So,

$$E(\lambda)d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda$$