1-4-2 Atom Stability

According to the concepts of classical physics, the atom is unstable system, because the electron changes the direction of its motion while it being rotate around the nucleus and therefore it is an accelerated charge. Since, the classical electromagnetic theory states that an accelerated charged particle emits electromagnetic energy. Therefore, the electron gradually loses its energy due to the emission and thus it is gradually approach from of the nucleus $(-\frac{1}{4\pi\epsilon_0}.\frac{e^2}{2r})$. The trajectory form of approaching is a spiral path continue steadily until it collide with the nucleus and consequently the atom is unstable system. Nevertheless, this is not the real situation because all evidence emphasizes that nucleus is a stable system that does not emit radiation under normal conditions, unless there is an external stimulation such as temperature. Thus, classical physics has once again failed to explain the stability of the atom.

In the early of 1900, Bohr succeeded in overcoming this problem using quantitative ideas that he assumed the following;

1- The angular momentum (L) of the electron revolving around the nucleus is an integer multiplied by a constant value symbolized by the symbol ħ equal to $h/2$. i.e.

$$L = n\hbar$$

2- The atom emits radiation when the electron jumps from orbit of energy E_1 to another orbit of lower energy E_2 with frequency given by;

$$\upsilon_{12}=(E_2 - E_1)/h$$

According to the first Boher hypothesis the angular momentum can written as follows;

$$L_n = m\, r_n^{\,2}\, \omega_n = n\, \hbar \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1\text{-}6)$$

Where r_n is the orbit radius of n and ω_n is the angular frequency of the electron in that orbit.

Coulomb force must be weighted centrifugal force so as an electron remain stable in an orbit. Therefore, for a particular atom like hydrogen the following relationship can be set up;

$$\frac{1}{4\pi\epsilon_0}.\frac{e^2}{r_n^2} = m w_n^2 r_n \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1\text{-}7)$$

When equation (1-6) solved relative to ω_n and the result is substituted in equation (1-7) the following expression can be get;

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{me^2}\, n^2 \qquad \text{………………..(1-8)}$$

Consequently;

$$w_n = \frac{me^4}{16\pi^2\epsilon_0^2\hbar^3}\,\frac{1}{n^3} \qquad \text{…………………(1-9)}$$

From equations (1-8 and 9) it can be seen that the electron can take an infinite number of orbits each of which is distinguished by the quantum number *n*.

Let us now try to find the mathematical formula for the total energy of the electron in the hydrogen atom for an orbit with a quantum number (*n*). It is well known that, the total energy of the electron is the sum of its kinetic ($\frac{1}{2} m\, v^2$) and potential energy ($-\frac{1}{4\pi\epsilon_0}.\frac{e^2}{r}$) respectively. Using equations (1-8) and (1-9) the potential and kinetic energy can write respectively as follows: -

$$V_n = -\frac{me^4}{16\pi^2\epsilon_0^2\hbar^2}\,\frac{1}{n^2} \qquad \text{…………………(1-10)}$$

$$T_n = \frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}\,\frac{1}{n^2} \qquad \text{…………………(1-11)}$$

Therefore;

$$E_n = T_n + V_n = -\frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2}$$

According to Bohr's second hypothesis, the loss of electron energy when jumping from orbit n_1 to n_2 is:

$$\Delta E_{12} = E_2 - E_1$$

$$\Delta E_{12} = \frac{me^4}{32\pi^2\epsilon_0^2\hbar^2}\left(\frac{1}{{n_2}^2} - \frac{1}{{n_1}^2}\right) \qquad \text{…………………..(1-12)}$$

Thus the frequency of emitted energy is;

$$\nu_{12} = \frac{me^4}{8\epsilon_0^2 h^3}\left(\frac{1}{{n_2}^2} - \frac{1}{{n_1}^2}\right) \qquad \text{…………………..(1-13)}$$

The wave number that is define as the number of waves per unit length can written as follows;

$$\bar{\nu}_{12} = \frac{\nu_{12}}{c} = \frac{1}{\lambda_{12}} = \frac{me^4}{8\epsilon_0^2 ch^3}\left(\frac{1}{{n_2}^2} - \frac{1}{{n_1}^2}\right) \qquad \text{………………..(1-14)}$$

The quantity $\frac{me^4}{8\epsilon_0^2 ch^3}$ called Rydberg constant and denoted by R_H.

1-4-3 Photoelectric Effect

The photoelectric phenomenon is the process by means the electrons are release from metals due to the electromagnetic radiation. The emitted electrons by this way called photoelectrons. This phenomenon characterized by the following:

a) There is a frequency value called threshold frequency (υ_o) which below this value no emission occurs, no matter how much the radiation intensity is.

b) The photoelectrons have a kinetic energy equal to E_r-W_o where E_r is the energy of the incident radiation and W_o the energy needed to separate electron from the metal called work function.

c) Some of the photoelectrons leave the metal surface after a period of time around 8-10 sec.

Again, the classical physics failed to give a convincing explanation for this phenomenon where: -

1 – It could not explain the threshold frequency.

2 – It states that the electrons leave the surface of the metal with kinetic energy equal to zero.

3 – It shown that the time required for absorbing the energy by photoelectrons is about 10 sec.

In 1905 Einstein was interpret this phenomenon by assuming that the radiation consists of particles of rest mass equal to call the photons. These photons behave with electrons in a way similar to the behavior of atomic oscillators with blackbody radiation. Furthermore, the energy of a single photon is ($E = h\upsilon = \hbar\omega$) and photoelectrons emitted because of collisions between photons and electrons. Therefore, the total energy of a single electron can write as follows;

$$h\nu = W_o + \frac{1}{2}mv^2$$

or; ……………………(1-15)

$$h\nu = h\nu_o + \frac{1}{2}mv^2$$

Equation (1-15) called Einstein's equation that explained the photoelectric phenomenon exactly. From this formula one may concluded the following;

1- Photonic electrons are emitted only if the photon energy is greater than W_o (i.e. $\upsilon > \upsilon_o$).

2- If the incident photon frequency υ is greater than υ_o, the photoelectrons emits with a kinetic energy equal to ($h\upsilon$ - W_o).

1-5 Natural Duality of Radiation and Particle

I. Duality of radiation means that radiation behave as a wave in certain phenomena, such as interference and interference, while acting as a particle in other phenomena such as the photoelectric phenomenon.

II. De Broglie has assumed that there is a wave associates with any moving particle with a velocity (v), and so its wavelength given by the relationship;

$$\lambda = \frac{h}{mv} = \frac{h}{p} \qquad \text{................ (1-16)}$$

Where m and p are the mass and linear momentum of the particle. So, as long as the hypothesis of de Broglie is valid, it shows that the particle behaves as a wave and thus has a dual nature.

Example: By using equations (1-6) and (1-16), show that the circumference length orbits in Bohr model of hydrogen atom given by:

$$2\pi r_n = n\lambda \qquad \text{.............. (1-17)}$$

Solution:

$$L = mv\, r_n = n\hbar$$
$$= n\, h / 2\pi$$

$\Longrightarrow$

$$2\pi r_n = n\, h / mv$$
$$= n\, h / p$$
$$= n\lambda$$

$$\therefore \; 2\pi r_n = n\lambda$$

1-6 Principle Heisenberg Uncertainty

This principle states that it is not possible to determine accurately at the same time special binary values of physical variables that describe the behavior of an atomic system. Furthermore, the amount of the multiplication of the uncertainties of these variables values at least equal to the quantity $\hbar$. Thus, mathematically we have;

$$\Delta a \,.\, \Delta b \;\geq\; \hbar$$

Where Δa is the uncertainty in determining the variable A and Δb is the uncertainty in finding the variable B.

Examples of these special pair of variable are the x-axis of the particle and its corresponding linear momentum (p_x), the angular momentum of the particle (L_z) and its angular position (φ), the energy (E) of the particle and the time t at which the measurement is done. Accordingly;

$$\left.\begin{array}{ll} \Delta x \,.\, \Delta p_x \;\geq\; \hbar & \ldots \text{(a)} \\ \Delta \varphi \,.\, \Delta L_x \;\geq\; \hbar & \ldots \text{(b)} \\ \Delta E \,.\, \Delta \text{t} \;\geq\; \hbar & \ldots \text{(c)} \end{array}\right\} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1-18)}$$

The physical meaning of the equations (1-18a) is as follows; the linear momentum component of a particle (p_x) cannot be finely determined without losing all the information about its position and vice versa. The value of the uncertainty in assigning the values of these variables is at least equal to ($\hbar$).

<u>H.W</u>

1- What is the physical meaning of equations (1-18b) and (1-18c)?

2-Use any method to drive equation (1-18a).

3-Use equation (1-18a) to obtain equation (1-18b).

1-7 Correspondence Principle _

This principle, which assumed by Bohr, states that the results of quantum physics are consistent with the results of classical physics at the limit where the Planck constant can be neglected.

Example: Use the uncertainty principle to prove the impossibility of finding electron inside a nucleus.

Solution:

Since the diameter of the nucleus is about 10^{-14}m, it means that the uncertainty in the determination of the location of the electron inside the nucleus (for presumable) does not exceed the diameter of the nucleus, i.e. $\Delta x \approx 10^{-14}\,\mathrm{m}$. However, since

$$\Delta x\, \Delta p \geq \hbar$$

Thus;

$$\Delta p \geq \frac{\hbar}{\Delta x} = \frac{1.054\times10^{-34}}{10^{-14}} = 1.054\times10^{-20}\ \ \mathrm{kg\,m\,s^{-1}}$$

$$E = \sqrt{p^2c^2 + m_\circ^2 c^4}$$

$$p^2c^2 >> m_\circ^2 c^4$$

$$\therefore \quad E = p\,c$$

$$\therefore \quad \Delta E = \Delta p\, c$$

$$1.05\times10^{-20} \times 3\times10^{8} \approx 20\mathrm{MeV}$$

Since the average binding energy inside the nucleus is estimated to be $8\mathrm{MeV}$, and this value is much less than the energy of the electron $20\mathrm{MeV}$, so the presence of electron inside the nucleus is not possible.