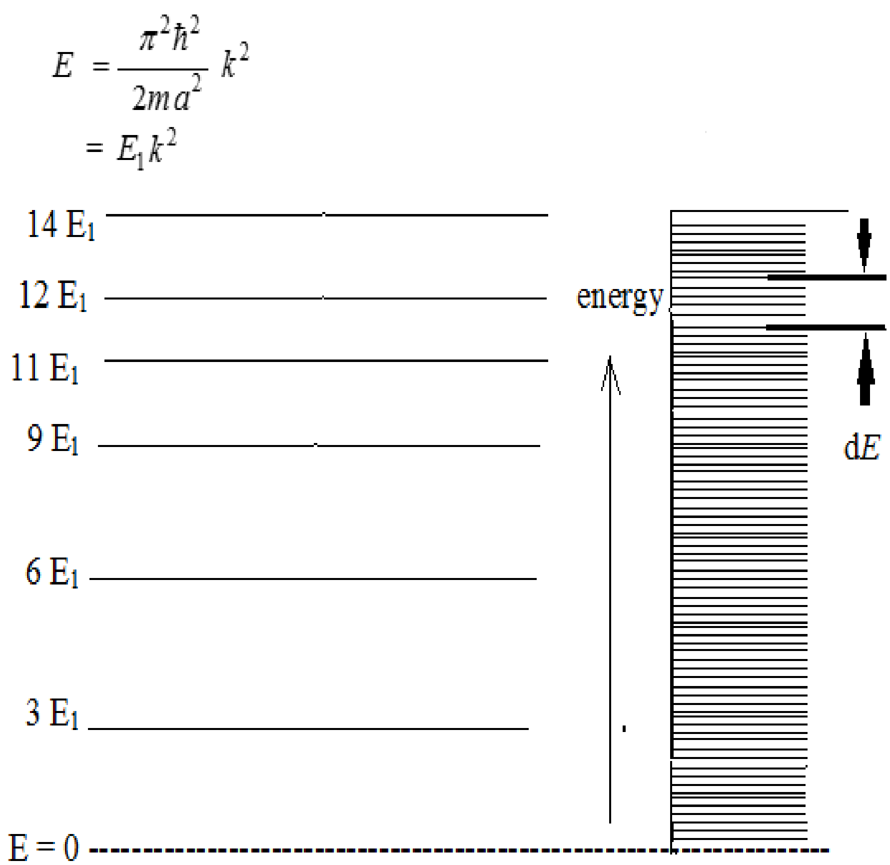
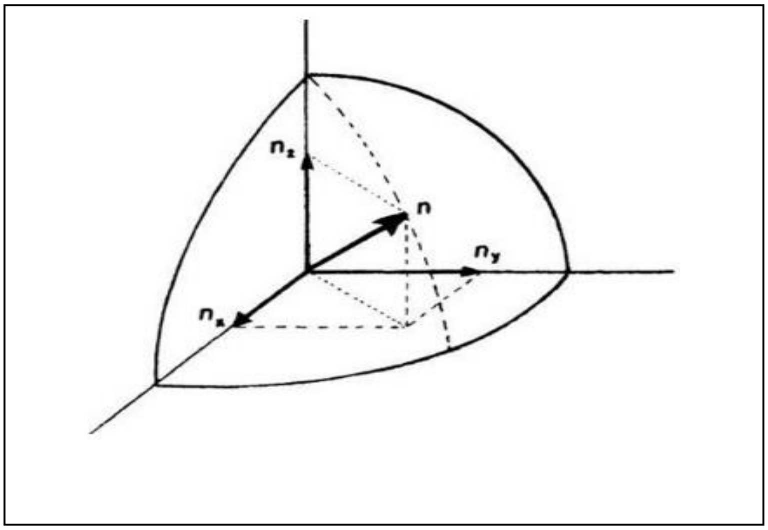
3-6 Density of states

Concerning with equation (3-34) it is obvious that the energy interval between any two-adjecint states depend on the widgth of the potential cube. Consequently, these states become very much closes to each other and hence indistinguishable as the box widgth increases. Strictly speaking, this means that as the size of of the box leaves the microscopic scale their dynamical variables becomes of a continuum values.



Anyway, the problem now is to find the number of energy levels for a range dE of the energy when the potential box being very large. Actually, the number of states (energy levels) that have enegy vales beween 0 to E (0 to k) can be deduces with aid of the figure below to be; $\frac{1}{8}\left(\frac{4}{3}\,\pi n^3\right)$, therefore;

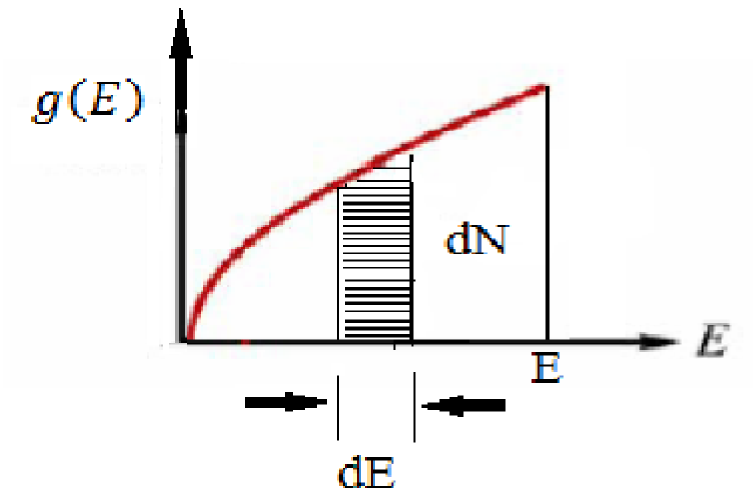


$$N(E) = \frac{8\pi V}{3h^3}(2m^3)^{\frac{1}{2}}E^{3/2} \qquad \text{…..……..(3-35)}$$

Where V = a^3 is the potential box volume. Consequently, the number of states (energy levels) with energy between *E* and *E*+*dE* is;

$$dN(E) = \frac{4\pi V(2m^3)^{\frac{1}{2}}}{h^3}E^{1/2}dE \qquad \text{…..……..(3-36)}$$

The quantity $\frac{4\pi V(2m^3)^{\frac{1}{2}}}{3h^3}E^{1/2}$ called ***density of states*** and defined as the number of states (energy levels) with energy between *E* and *E*+*dE* and mathematically written as; $g(E) = \frac{dN(E)}{dE}$. Howevere, the intgreation of the density states curve, see figure below, from *E* to *E*+ε gives the total number of states in that energy interval.



H.W: show that;

i- $\frac{dN(P)}{dP} = g(P) = g(E)\frac{dE}{dP} = \frac{4\pi V}{h^3}P^2$

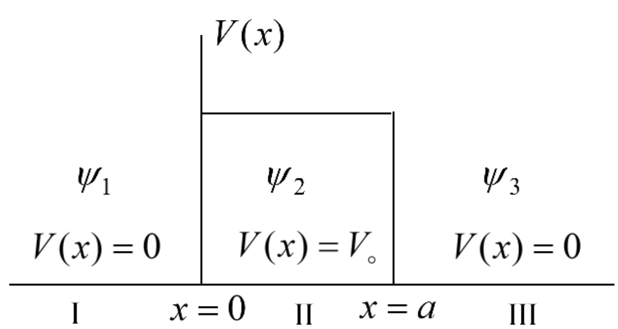
ii- $\frac{dN(\gamma)}{dP} = g(\gamma) = g(P)\ \frac{dP}{dv} = \frac{4\pi V}{c^3}\gamma^2$

3.7 Potential Barrier Penetration

The fact that the wave function can be continue extendes beyond the classically forbeden region give rise to expect that could be penetrate a potential berrier. So let us investigate a particle moves in the folloeing potential distribution.

$$V(x) = \begin{cases} 0 & x < 0 & region - I \\ V_o & 0 \le x \le a & region - II \\ 0 & x > a & region - III \end{cases}$$

Which graphically equvalint to;



a) $E < V_\circ$

Results of classical mechanics predicts that a particle of energy E less than the height of the barrier V_o definitely reflect back at x=0. While the correspondence one of quantum mechanics show different behavior as shown below;

i) Region-I

Schrodinger equation in this region takes the form;

$$\frac{d^2\psi_1}{dx^2} + \frac{2mE}{\hbar^2}\psi_1 = 0, \text{ or}$$

$$\frac{d^2\psi_1(x)}{dx^2} + k^2\psi_1(x) = 0$$

Where $k^2 = \frac{2mE}{\hbar^2}$. The solution of this equation is;

$$\psi_1(x) = Ae^{ikx} + Be^{-ikx} \qquad \text{……………(3-37)}$$

The inclusions obtained before are applicable here, where the first term in last equation refers to the wave function of the incident particle while the second one represent the wave function of the particles that are reflected back at $x=0$.

ii)Region-II

Schrodinger equation for this case has the form;

$$\frac{d^2\psi_2}{dx^2} - \frac{2m}{\hbar^2}(V_\circ - E)\ \psi_2 = 0, \text{ or}$$

$$\frac{d^2\psi_2}{dx^2} - \alpha^2\psi_2 = 0$$

Where $\alpha^2 = \frac{2m}{\hbar^2}(V_\circ - E)$. The solution of this equation is;

$$\psi_2(x) = Ce^{-\alpha x} + De^{+\alpha x} \qquad \text{……………(3-38)}$$

The second term cannot be ignored as before due the end edge of the potential at $x=a$.

H.W: Verify whether the wave function in equation (3-38) satisfies the boundary condition or not.

iii) Region-III

Schrodinger equation in this region takes the form;

$$\frac{d^2\psi_3}{dx^2} + \frac{2mE}{\hbar^2}\psi_3 = 0, \text{ or}$$

$$\frac{d^2\psi_3(x)}{dx^2} + k^2\psi_3(x) = 0$$

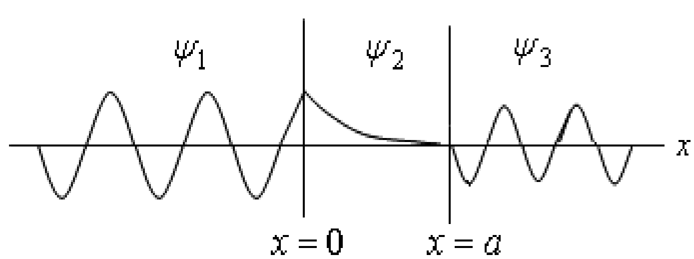
Where $k^2 = \frac{2mE}{\hbar^2}$. The solution of this equation is;

$$\psi_3(x) = A'e^{ikx} + B'e^{-ikx}$$

The second term of the last equation must be ignore because there is no reasons makes the wave function (particle)) reflected back in this region. Thus;

$$\psi_3(x) = A'e^{ikx} \qquad \text{……………. (3-37)}$$

The last equation shows that it is possible for the incident particle to go through the potential berrier even if its kinetic energy is less than the height of the barreier. Furthermore, the trassmiited particle has energy equal to its incident one but has a less probability to be in region-III. i.e $A > \acute{A}$, see the figure below.



b) E > V_o

Results of the classical mechanics for this case indicate that all of the incident particles on the potential barrier pentrates the potential barrier and transmitted to the third region. While results of quantum mechanics shows a such possibility of some of the particles to reflectes back at the two points $x = a$, $x = 0$, as will be shown below.

i) Region-I

Schrodinger equation for this case being as follows;

$$\frac{d^2\psi_1(x)}{dx^2} + k^2\psi_1(x) = 0$$

Where $k^2 = \frac{2mE}{\hbar^2}$. Solution of this equation given by;

$$\psi_1(x) = Ae^{ikx} + Be^{-ikx} \qquad \text{……………. (3-38)}$$

ii) Region-II

Schrodinger equation for this case becomes;

$$\frac{d^2\psi_2}{dx^2} + k'^2\psi_2 = 0$$

Where $k'^2 = \frac{2m}{\hbar^2}(E - V_\circ)$. Solution of this equation is;

$$\psi_2(x) = Ce^{ik'x} + De^{-ik'x} \qquad \text{……………. (3-39)}$$

iii) Region-III

Schrodinger equation for this case being as follows;

$$\frac{d^2\psi_3}{dx^2} + \frac{2mE}{\hbar^2}\psi_3 = 0$$

Where $k^2 = \frac{2mE}{\hbar^2}$. Solution of this equation given by;

$$\psi_3(x) = A'e^{ikx} + B'e^{-ikx}$$

The second term in the last equation must be neglects for the same reasons mentioned previously. Thus;

$$\psi_3(x) = A'e^{ikx} \qquad \text{……………. (3-40)}$$