**Lecture 6**

**S 1.4 : Unit Circle and Basic Trigonometric Functions**

**Definition 1:** Let ***x*** be any real number and let *U* be the unit circle

with equation  ( the centre of the circle *U* is the point O (0,0) ,

and the radius of the circle *U* equals 1 ) . Start from the point A(1,0) on

*U* and proceed counterclockwise if  is positive and clockwise if

is negative around the unit circle  *U* until an arc length of has

been covered . Let P(***a* , *b***) be the point at the terminal end of the arc .

The measurement of the angle AOP is radians .

 If radians = ( degrees ) ,

 then the following six

 trigonometric functions of

 are defined in terms of the

 coordinates of the circular

 point P(***a , b***) :

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  *b*

 **P**(*a*, *b*)

 *units*

 **A**(1,0)

 **O**(0,0)

 ●

 ●

 *radians*

  *a*

**1)  = *b* =  ( *x* radians) **

**2) =  *a* =  ( *x* radians) **

**3) =  **

 **=  ( *x* radians) **

**4) =  **

 **=  ( *x* radians) **

**5)  =  **

 **= ( *x* radians) **

**6)  =  **

 **=  ( *x* radians) **

**Remark 1:** Definition 1 uses the standard function notation , , with *f* replaced by the name of a particular trigonometric function . For example ,  actually means  and  actually means  .

**Remark 2:** Remember that  and

 .

**Theorem 1:**

For any real number *x* we have the following trigonometric identities :

**1)  .**

**2)  .**

**3)  .**

**4)  .**

**5)  .**

**6)  .**

**7)  .**

**8)  .**

**9)  .**

**10)  .**

**11) .**

**12) .**

**S 1.5: Graphs of Sine and Cosine Functions**

**1.5.1: Table for values of sin *x* , cos *x* , and tan *x* for selected values**

**of *x***

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Values of*****x*** | **Degrees** | **0** | **30** | **45** | **60** | **90** |
| **Radians** | **0** |  |  |  |  |
| **sin *x*** | **0** |  |  |  | **1** |
| **cos *x*** | **1** |  |  |  | **0** |
| **tan *x*** | **0** |  | **1** |  |  **Undefined** |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Values of*****x*** | **Degrees** | **120** | **135** |  **150** | **180** | **270** |
| **Radians** |  |  |  |  |  |
| **sin *x*** |  |  |  |  **0** |  |
| **cos *x*** |  |  |  |  | **0** |
| **tan *x*** |  |  |  |  **0** |  **Undefined** |

**Definition:** A function *f* is periodic if there exists a positive real number

 *p* such that  for all *x* in the domain of *f* .

The smallest such positive number *p*  is the period of *f .*

**Remarks :**

1. The functions sin *x* , cos *x* , sec *x* , and csc *x* are periodic functions

 with period 2 .

2) The functions tan *x* and cot *x* are periodic functions with period  .

**1.5.2: The Graph of sin *x***

The graph of the function *y* = sin *x* is the line passing through all the points ( *x* ,sin *x*) on the  *x y* - plane .

 The graph of the function *y* = sin *x* for the interval  is the line

passing through the points ( 0 , 0 ), ,  ,  , ,

 ,  ,  , and  which is shown in the

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**● ● ●**

 **● ●**

 **●**

following figure

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The graph of the function *y* = sin *x* is shown in the following figure

 

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The period of the function  *y* = sin *x* is 2 . The domain of the function

 *y* = sin *x* is the set of all real numbers R .

The range of the function *y* = sin *x* is the interval ** .**

**1.5.3: The Graph of cos *x***

The graph of the function ***y*** = **cos *x*** is the line passing through all

the points **( *x* ,cos *x*)** on the  *x y* - plane .

The graph of the function *y* = cos *x* for the interval  is the line

Passing through the points **(** 0 **,** 1 **), ,  ,  , ,  ,  , ** , and  which is shown in the

following figure

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The graph of the function *y* = cos *x* is shown in the following figure



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The period of the function *y* = cos *x* is 2 .

The domain of the function *y* = cos *x* is the set of all real numbers R .

The range of the function *y* = cos *x* is the interval  .

**1.5.4: The Graphs of tan *x* and sec *x***

The graph of the function **** is the line passing through all

the points **( *x* ,tan *x*)** on the  *x y* - plane .

The graph of  is shown in the following figure

 ****

**0**

***x-axis***

 ***y-axis***

**The graph of  is shown in the following figure**

***y-axis***

 ****

**0**

***x-axis***

**Exercise: Draw the graph of** **the following** **trigonometric functions :**



