Definition:-

Let X be a Set. A Function $f: J \longrightarrow X$ is called sequnce in X.

Remarks:-

1. For each $n \in J$ we denoted by the value $f(n)$ by $\{a_n\}$ or $\langle a_n \rangle$.
2. if $X = \mathbb{R}$, Then the sequnce is called the sequnee of real numbers.
3. if X is Countable then the range of the sequnce is X.

Theorem :-

Every infinite subset of countable set is countable.

proof :- Let X be a countable set

and A be an infinite subset of X.

$\Rightarrow X \sim J$

and $J \sim X$ (by def. of countable set)

Then X is a range of sequnce.

element of X can be written $x_1, x_2, ----, --$ which are distance element.

Let n_1 be the smallest positive integer $\ni x_{n_1} \in A$.

" n_2 " " " " " $\ni n_2 > n_1$ and $x_{n_2} \in A$

" n_3 " " " " " $\ni n_3 > n_2$ and $x_{n_3} \in A$.

⋮ ⋮

n_k " " " " " $\ni n_k > n_{k-1}$ and $x_{n_k} \in A$.

Let $f : J \longrightarrow A$ defined by $f(k) = x_{n_k}$.

$\therefore f$ is 1-1 and onto.

$\therefore J \sim A \Rightarrow A \sim J$.

$\therefore A$ is countable set.

Theorem :- let $\langle E_n \rangle$ be a sequnce of countable set and let $S = \bigcup\limits_{n=1}^{\infty} E_n$, then S is countable.

proof :- since E_n is countable for each n.

$\therefore J \sim E_n$ for each n.

elements of E_n can be arranged is a sequnce of distant element d.