

سعيه "B" مجموعة "A"

Q.3 Solve the ~~the~~ initial-value problem  
 $y^2 dx + (3xy + y^2 - 1) dy = 0$

with  $y(1) = -1$

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الطريقة الاولى :- اعتبار هذه الدالة (Non exact)

$$M = y^2 \Rightarrow M_y = 2y$$

$$N = 3xy + y^2 - 1 \Rightarrow N_x = 3y$$

$M_y \neq N_x$  non exact

$$\frac{N_x - M_y}{N} = \frac{3y - 2y}{y^2} = \frac{1}{y}$$

$$I = e^{\int \frac{1}{y} dy} = e^{\ln y} = y$$

$$I[M dx + N dy = 0] \Rightarrow y[y^2 dx + (3xy + y^2 - 1) dy = 0]$$

$$I.M = y^3 \Rightarrow (I.M)_y = 3y^2$$

$$I.N = 3xy^2 + y^3 - y \Rightarrow (I.N)_x = 3y^2$$

$$(I.M)_y = (I.N)_x \Rightarrow \therefore \text{exact}$$

$$F(x, y) = \int (I.M) dx = xy^3 + h(y) = C$$

$$\frac{\partial F}{\partial y} = 3xy^2 + h'(y) = I.N = 3xy^2 + y^3 - y$$
$$h'(y) = y^3 - y \Rightarrow h(y) = \frac{1}{4}y^4 - \frac{1}{2}y^2 + C$$

$$\therefore F(x, y) = xy^3 + \frac{1}{4}y^4 - \frac{1}{2}y^2 = a, \quad a = C - C_1$$

$$\Rightarrow (1)(-1) + \frac{1}{4} - \frac{1}{2} = a \Rightarrow a = -\frac{5}{4}$$