

Q.2 Find the orth. traj. of a family of ellipses with center (0,0) and vertices (0,1), (0,-1).

The form of ellipse equation is:-

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

$$\Rightarrow \frac{y^2}{a^2} + \frac{x^2}{b^2} = 1 \Rightarrow \boxed{\frac{y^2}{1} + \frac{x^2}{b^2} = 1}$$

$$\Rightarrow \frac{x^2}{b^2} = 1 - y^2 \Rightarrow b^2 = \frac{x^2}{1 - y^2}$$

$$\Rightarrow 2yy' + \frac{2x}{b^2} = 0 \Rightarrow \frac{2x}{b^2} = -2yy'$$

$$y' = \frac{-x}{yb^2} = \frac{-x}{y \frac{x^2}{1-y^2}} = \frac{-(1-y^2)}{xy}$$

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$$\Rightarrow \frac{dy}{dx} = \frac{xy}{(1-y^2)} \quad \text{فصل متغيرات}$$

$$\int \frac{(1-y^2) dy}{y} = \int x dx \Rightarrow \int \frac{dy}{y} - \int y = \int x$$

$$\ln y - \frac{1}{2} y^2 = x + C$$