شعبة "B" مجموعة "B"

Q.1 Solve The I.V.B

$$\frac{dy}{dx} + 2xy = y^2 e^{x^2} \ln x \quad , \text{with } y(1) = 1$$

Sol.// The equation is Ber. diff. eq.

$$\left\{ \frac{dy}{dx} + 2xy = y^2 e^{x^2} \ln x \right] * \frac{1}{y^2}$$

$$y^{-2} \frac{dy}{dx} + 2x y^{-1} = e^{x^2} \ln x$$

let $z = y^{-1}$, $\frac{dz}{dx} = -y^{-2} \frac{dy}{dx}$

$$\left\{ -\frac{dz}{dx} + 2xz = e^{x^2} \ln x \right] * (-1)$$

$$\frac{dz}{dx} - 2xz = -e^{x^2} \ln x$$ linear diff. eq.

$$P(x) = -2x \;;\; Q(x) = -e^{x^2} \ln x$$

$$I = e^{\int -2x\,dx} = e^{-x^2}$$

$$\Rightarrow e^{-x^2} \cdot z = \int \cancel{e^{-x^2}}\, \cancel{e^{x^2}} \ln dx$$

$\int \ln x \, dx \Rightarrow$ let $u = \ln x$, $du = \frac{1}{x} dx$
$dv = dx$, $v = x$

$$e^{-x^2} \cdot z = -x \ln x + \int \frac{1}{\cancel{x}} \cancel{x}\, dx = -x \ln x + x + c$$

$$\Rightarrow \frac{1}{y} = \frac{-x \ln x}{e^{-x^2}} + \frac{x}{e^{-x^2}} + \frac{c}{e^{-x^2}} \Rightarrow 1 = \frac{-(1)\ln 1}{e^{-1}} + \frac{1}{e^{-1}} + \frac{c}{e^{-1}}$$

$$\Rightarrow c = (e^{-1} - 1) \Rightarrow \frac{1}{y} = -x \ln x + x + (e^{-1} - 1)$$