

Q1 Find the I.V.P.

$$(2y + \tan x) dy + (y \sec^2 x + \sec x \tan x) dx = 0$$

with $y(\frac{\pi}{4}) = 1$

Solⁿ $M dx + N dy = 0$

The eq is ~~is exact~~

$$M = y \sec^2 x + \sec x \tan x, \quad M_y = \sec^2 x$$

$$N = 2y + \tan x, \quad N_x = \sec^2 x$$

$M_y = N_x$ \therefore The eq. is exact

$$F(x, y) = \int M dx = \int (y \sec^2 x + \sec x \tan x) dx + c$$
$$= y \tan x + \sec x + h(y) = c$$

$$\frac{\partial F}{\partial y} = \tan x + h'(y) = N$$
$$= \tan x + h'(y) = \tan x + 2y$$

$$h(y) = y^2 + a$$

$$h(y) = y^2 + a = C^*$$

$$F(x, y) = y \tan x + \sec x + y^2 = C$$
$$\Rightarrow (1) \tan(\frac{\pi}{4}) + \sec(\frac{\pi}{4}) + (1) = C$$

$$\Rightarrow 1 + \sqrt{2} + 1 = C \Rightarrow 2 + \sqrt{2}$$

$$F(x, y) = y \tan x + \sec x + y^2 = (2 + \sqrt{2})$$