"D"

شعبة "C" مجموعة "A"

Q.1 Solve the I.V.P.

$3x^2y\,dx + (-x^3 - 2y^4)\,dy = 0$ with $y(0) = 1$

Sol.// $M\,dx + N\,dy = 0$.

$M = 3x^2y$, $M_y = 3x^2$

$N = -x^3 - 2y^4$, $N_x = -3x^2$

$M_y \neq N_x$ ∴ is non exact diff. eq.

$$\frac{N_x - M_y}{M} = \frac{-3x^2 - 3x^2}{3x^2y} = \frac{-6x^2}{3x^2y} = \frac{-2}{y}$$

$$I = e^{\int -\frac{2}{y}dy} = e^{-2\ln y} = \frac{1}{y^2}$$

$$\frac{1}{y^2}[3x^2y\,dx + (-x^3 - 2y^4)]dy = 0$$

$M.I = \frac{3x^2}{y}$ ⇒ $(M.I)_y = \frac{-3x^2}{y^2}$

$N.I = \frac{1}{y^2}(-x^3 - 2y^4)$ ⇒ $(N.I)_x = \frac{-3x^2}{y^2}$

$\int (M.I)\,dx = F(x,y)$

$F(x,y) = \frac{x^3}{y} + h(y) = C$

$\frac{\partial F}{\partial y} = -\frac{x^3}{y^2} + h'(y) = N.I = -\frac{x^3}{y^2} - 2y^4$

$h(y) = -\frac{2}{5}y^5 + a$

$F(x,y) = \frac{x^3}{y} - \frac{2}{5}y^5 = C_*$, $C_* = \frac{0}{1} - 1 = -1$

$F(x,y) = \frac{x^3}{y} - \frac{2}{5}y^5 + 1$