"B" مجموعة ، "C" شعبة

Q. Solve the initial value problem
$(x\cos x)\frac{dy}{dx} + y(x\sin x + \cos x) = 1$ with $y(\pi) =$

Sol.//

$$\left[(x\cos x)\frac{dy}{dx} + y(x\sin x + \cos x) = 1\right] * \frac{1}{x\cos x}$$

$$\frac{dy}{dx} + y\,\frac{x\sin x + \cos x}{x\cos x} = \frac{1}{x\cos x}$$ linear d.e

$$\frac{dy}{dx} + y\left(\frac{\sin x}{\cos x} + \frac{1}{x}\right) = \frac{1}{x\cos x}$$

$$I = e^{\int\left(\frac{\sin x}{\cos x} + \frac{1}{x}\right)dx} = e^{-\ln|\cos x| + \ln x}$$

$$= \frac{1}{\cos x}\cdot x = \frac{x}{\cos x} = x\sec x$$

$$I \cdot y = \int I \cdot Q(x)$$

$$x\sec x \cdot y = \int x\sec x \cdot \frac{1}{x\cos x}\,dx$$

$$x\sec x \cdot y = \int \sec^2 x\,dx = \tan x + C$$

$$\pi(\sec\pi)\cdot 1 = \tan\pi + C$$

$$\pi(-1) = 0 + C \Rightarrow C = -\pi$$