Q/ Solve the I.V.P. "C" as يجي "C" يطلع

$$\frac{dy}{dx} - 2y\tan x = y^2\tan^2 x$$

with $y(\frac{\pi}{4}) = 1$

Sol// The equation is Bernoulli diff. eq.

$$\left\{\frac{dy}{dx} - 2y\tan x = y^2\tan^2 x\right\} * \frac{1}{y^2}$$

$$y^{-2}\frac{dy}{dx} - 2y^{-1}\tan x = \tan^2 x$$

let $z = y^{-1}$, $\frac{dz}{dx} = -y^{-2}\frac{dy}{dx}$

$$\left\{-\frac{dz}{dx} - 2z\tan x = \tan^2 x\right\} * (-1)$$

$$\frac{dz}{dx} + 2z\tan x = -\tan^2 x$$

linear diff. eq. w.r.t. (z)

$P(x) = 2\tan x$, $Q(x) = -\tan^2 x$

$$I = e^{\int 2\tan x\,dx} = e^{-2\ln|\cos x|} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$\int \sec^2 x \cdot z = -\int \tan^2 x \cdot \sec^2 x\,dx$$

$$= -\frac{\tan^3 x}{3} + C$$

$$z = -\frac{\sin^3 x}{3\cos x} + C(\cos^2 x)$$

$$\frac{1}{y} = \frac{-\sin^3 x}{3\cos x} + C(\cos^2 x)$$