Lecture Seven **:The basic rules and identities of Boolean algebra :**

**a-** x.0=0 **b-** x+0=x **c-** x.1=x **d-** x+1=1

**e-** x.x=x **f-** x+x=x **g-**  x.x =0 **h-**  x+x=1

**i-**  x=x

**2-Commutative laws :**

**a-** x.y=y.x

**b-** x+y=y+x

**3-Associative laws :**

**a-** (x+y)+z = x+(y+z) = x+y+z

**b-** (x.y).z = x.(y.z) = x.y.z

**4-Distributive laws :**

**a-**  x.(y+z) = (x.y)+(x.z)

**b-** x+(y.z) = (x+y).(x+Z)

**5-Absorption laws :**

**a-** x+x.y = x

**b-** x+x.y = x+y

**c-**  x.(x+y) = x

**6-De Morgan´s theorem :**

**a-**  x.y = x+y

**b-** x+y = x.y

All of these Boolean theorems useful in simplifying a logic expression that is in reducing the no. of terms in the expression .

***EX***: Prove that A.B = A+B

Sol: A.B = A+B = A+B

***EX***: Simplify the following :

Z=A(A+B)

Sol: Z = AA+AB = A+AB = A

***EX***: write the Boolean exp. For the following circuit

A

B

C Z

Sol: Z = (A+B).C

***EX***: write the Boolean exp. For the following logic cct. :

A

Z

B

Sol : Z = AB+AB = A ⊕ B

***EX***: write Boolean exp. For the following logic cct. :

A

B

Z

C

Sol: Z = (AB+AB).C =(A ⊕ B).C

**NOTE :** The priorities of logic operation are :

( )

AND

OR

***EX***: Construct the logic circuit for the following Boolean exp. :

Z = A.B+B

Sol:

We need 2-input AND gate A

B  2-input OR gate

2-inverters Z

***EX***: Construct the logic cct. , and write the T.T. for the following logic eq.

Z = AC+ABC

ABC Z

Sol: 000 0

A  001 0

C  Z 010 0

011 0

B 100 0

101 1

110 0

111 1

***Ex*:** simplify the following :

A= x y z + x y z + x y z + x y z =x z (y+y) + x z (y+y) = x z +x z = z (x+x) = z

***Ex:*** simplify the following :

Z= AB + A(B+C) + B(B+C)

= AB + AB + AC + BB + BC

= AB + AC + B + BC

= AB + AC + B

= B + AC

***Ex:*** Simplify the following :

Z = ABC + ABC + ABC

= ABC + AC(B+B)

= ABC + AC

=C (AB+A)

=C (A+B)

=CA+CB

***Ex***: Simplify the following using Boolean Algebra

F= A{ BC(A+B+C+D)}

= ABC(A+B+C+D)

=ABCA +ABCB +ABCC +ABCD

=ABC+ABC+ABC+ABCD

=ABC+ABCD

=ABC(1+D)

= ABC

***EX***: Simplify:

F = AC + ABC + ACD + CD

= A(C+BC) + C(AD+D)

= A(C+B)+C(A+D)

= AC + AB + CA +CD

= A(C+C) + AB + CD

= A + AB +CD

= A(1+B) + CD

= A + CD

***EX***: Prove that :

F= ABC + ABC + ABC = A(B+C)

= AC(B+B) + ABC

= AC + ABC

= A(C+BC)

= A(C+B)

***EX***: Simplify :

X = AB + ABC + AB + ABC

= AB(1+C) + AB + ABC

= AB + AB + ABC

= B + ABC

= B + BAC

= B + AC

***EX***: In a 3-input cct. The output is (1) if the majority of input is (1) , and otherwise , it is zero. White the T.T. for this cct. :

ABC Z

000 0

001 0

010 0

011 1

100 0

101 1

110 1

111 1