

## On $K_{\alpha c}$ -spaces

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**Abstract.** That research is submitted to introduce new type of supra  $Kc$ -spaces, it is supra  $\alpha K$  ( $\alpha c$ )-space, supra  $L$  ( $\alpha c$ )-space, supra  $(\alpha L)c$ -space and supra  $\alpha L(\alpha c)$ -space. We showed the relation between these types, also we provided some theorems, propositions and examples about the subjects.

**Keywords:** supra  $\alpha K$  ( $\alpha c$ )-space, supra  $L(\alpha c)$ -space, supra  $(\alpha L)c$ -space, supra  $\alpha L(\alpha c)$ -space, supra  $\alpha$ -Lindelöf space.

### 1. Introduction

Wilansky was the first author presented the notion of  $Kc$ -spaces [1], after that many authors treated with this space such as in [2]. Mashhour in [3] provided the notion of supra spaces. In [3], [4] the researchers presented the definition of the supra closure and the supra interior to any set in the supra space. In [5] the researcher defined the supra continuous function as (whenever  $g^{-1}(V)$  is supra open subset of the supra space  $(X, \mu_X)$  to any supra open subset  $V$  of the supra space  $(Y, \mu_Y)$ , then  $g : (X, \mu_X) \rightarrow (Y, \mu_Y)$  is supra continuous function). The researcher in [6] introduced the definition of supra open function

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as  $(g : (X, \mu_X) \rightarrow (Y, \mu_Y))$  is supra open function, whenever for any supra open subset  $S$  of  $X$ ,  $g(S)$  is supra open subset of  $Y$ . As well as he introduced the definition of the supra homeomorphism functions. In [7] Mukherji and Sarkar presented the concept of Lc-space (when any Lindelöf subset of a space  $X$  is closed, thus  $X$  is Lc-space). After that [8] the researchers introduced a kind of Lc-space which is stronger than the Kc-space. In this our work we connect between  $K(\alpha c)$ -space and the supra space. Also we connect  $\alpha K(\alpha c)$ -space and the supra space. By the same way we defined the rest Kind of spaces that we provided in this paper.

## 2. On $K(\alpha c)$ -spaces

In the next section of the research we will introduce a new types of supra Kc-spaces, we will begin with some definitions that are benefit us in our next topics.

**Definition 2.1** ([9]). Consider  $(X, \mu)$  is a supra space and  $S \subseteq X$ , when  $S \subseteq \text{Int}^\mu(\text{cl}^\mu(\text{int}^\mu)(S))$ , so  $S$  is supra  $\alpha$ -open set.  $S^c$  is called supra  $\alpha$ -closed.

**Definition 2.2** ([9]). A supra  $\alpha$ -open cover  $\{S_\alpha\}_{\alpha \in \Lambda}$  to  $(X, \mu)$  is a family of supra  $\alpha$ -open subsets of  $X$ , where  $X \subseteq \bigcup_{\alpha \in \Lambda} S_\alpha$

**Definition 2.3** ([9]). Whenever each supra  $\alpha$ -open cover for a supra space  $(X, \mu)$  owns a finite (countable) subcover, then  $X$  is supra  $\alpha$ -compact (supra  $\alpha$ -Lindelöf).

**Definition 2.4.** When each compact subset of  $(X, \mu)$  is supra  $\alpha$ -closed, so  $X$  is supra  $K(\alpha c)$ -space.

**Example 2.1.** The discrete supra space, is supra  $K(\alpha c)$ -space.

**Remark 2.1.** The supra  $\alpha$ -compact space is supra  $\alpha$ -Lindelöf.

**Example 2.2.** The supra co-countable space  $(R, \mu_{coc})$  is supra  $\alpha$ -Lindelöf, while it is not supra  $\alpha$ -compact.

**Definition 2.5** ([10]). If  $X \subseteq \bigcup_{\alpha \in \Lambda} U_{\alpha\alpha}$ , where  $\{U_\alpha\}_{\alpha \in \Lambda}$  is a family of supra open subsets from  $(X, \mu)$ , then  $\{U_\alpha\}_{\alpha \in \Lambda}$  is a supra open cover for  $X$ .

**Definition 2.6** ([10]). When each supra open cover to a supra space  $X$  owns a finite (or able to be counted) sub cover, then  $X$  is supra compact (supra Lindelöf) space.

**Remark 2.2** ([10]). Suppose  $X$  is Supra  $\alpha$ -compact (supra  $\alpha$ -Lindelöf) space, so it will be supra compact (supra Lindelöf).

**Example 2.3.** The discrete supra topology on infinite countable set is a supra Lindelöf space as well as a supra  $\alpha$ -Lindelöf space but neither supra compact nor supra  $\alpha$ -compact.

**Proposition 2.1** ([9]). *Each supra  $\alpha$ -closed subset from a supra  $\alpha$ -compact space will be supra  $\alpha$ -compact set.*

**Proposition 2.2** ([9]). *Suppose  $X$  is supra  $\alpha$ -Lindelöf space, and  $T$  is supra  $\alpha$ -closed subset of  $X$ , hence  $T$  is supra  $\alpha$ -Lindelöf.*

**Definition 2.7.** *Consider the supra space  $(X, \mu)$  and  $x, y$  is non-equal points in  $X$ , when we can find supra  $\alpha$ -open sets  $V_1$  in the supra space  $X$ , in which  $x \in V_1, y \notin V_1$ , so  $X$  is supra  $\alpha T_1$  space.*

**Example 2.4.** The supra co-finite topology on  $\mathbb{R}$  is supra  $\alpha T_1$ -space.

**Definition 2.8** ([11]). *Suppose  $(X, \mu)$  is a supra space,  $x, y$  are non-equal points in  $X$ , we can find are supra  $\alpha$ -open sets  $G_1, G_2$  in the supra space  $X$ , where  $x \in G_1, y \in G_2$  and  $G_1 \cap G_2 = \emptyset$  so  $X$  is supra  $\alpha T_2$ -space.*

**Example 2.5.** The discrete supra topology on  $\mathbb{R}$  is supra  $\alpha T_2$ -space.

**Theorem 2.1.** *Consider  $(X, \mu)$  is a supra  $\alpha T_2$ -space, and  $T$  is supra  $\alpha$ -compact subset of it, then  $T$  is a supra  $\alpha$ -closed.*

**Proof.** Take  $d \in T^c$  then for any point  $b \in T, d \neq b$ . There exist disjoint supra  $\alpha$ -open sets  $G, L$  in the supra space  $X$  containing  $b, d$  respectively. We get  $H = \{G_b\}_{b \in T}$  is supra  $\alpha$ -open cover to  $T$ , since  $T$  is supra  $\alpha$ -compact set, hence  $H$  owns a finite sub cover in which  $T \subseteq \cup_{i=1}^n G_{b_i}$ . Suppose  $\tilde{L} = \cap_{i=1}^n L_{b_i}$  which is supra  $\alpha$ -open set having  $d$  in it, implies  $\tilde{G} = \cup_{i=1}^n G_{b_i}$ , then  $\tilde{G} \cap \tilde{L} = \emptyset$ , so that  $T \cap \tilde{L} = \emptyset, \tilde{L} \subseteq T^c$ , therefore  $T^c$  is supra  $\alpha$ -open. Hence  $T$  will be supra  $\alpha$ -closed set.  $\square$

**Definition 2.9.** *Suppose  $T$  is a supra  $\alpha$ -compact subset of the supra space  $X$ , whenever  $T$  supra  $\alpha$ -closed, hence the space  $X$  is called supra  $\alpha K$  ( $\alpha c$ )-space.*

**Example 2.6.**

1-The co-countable supra space  $(\mathbb{R}, \mu_{coc})$  is supra  $\alpha K$  ( $\alpha c$ )-space, also it is  $K_c$ -space.

2-  $(\mathbb{R}, \mu_D)$  is supra  $\alpha K$  ( $\alpha c$ )-space.

**Proposition 2.3.** *Consider the supra spade  $(X, \mu)$  and  $T \subseteq X$ .  $T$  is supra  $\alpha$ -closed set iff there is a supra closed set  $L$  with  $cl^\mu(int^\mu(L)) \subseteq T \subseteq L$ .*

**Proof.** From  $cl^\mu(int^\mu(T)) \subseteq T (cl^\mu(T)) \subseteq T$  and  $\subseteq cl^\mu(T)$ , we have  $cl^\mu(int^\mu(cl^\mu(T))) \subseteq T \subseteq cl^\mu(T)$ , pick  $cl^\mu(T) = L$ , hence  $cl^\mu(int^\mu(L)) \subseteq T \subseteq L$ .

Conversely, since  $cl^\mu(int^\mu(L)) \subseteq T \subseteq L$ , so  $cl^\mu(cl^\mu(int^\mu(L))) \subseteq cl^\mu(T) \subseteq cl^\mu(L) = L$ , then  $cl^\mu(int^\mu(T))(L) \subseteq cl^\mu(T) \subseteq L$ , that implies,  $int^\mu(cl^\mu(int^\mu(L))) \subseteq int^\mu(cl^\mu(T)) \subseteq int^\mu(L)$ , then  $cl^\mu(int^\mu(cl^\mu(int^\mu(L)))) \subseteq cl^\mu(int^\mu(cl^\mu(T))) \subseteq cl^\mu(int^\mu(L))$  (because  $cl^\mu(int^\mu(L)) \subseteq T \subseteq L$ ), therefore  $cl^\mu(int^\mu(cl^\mu(T))) \subseteq T$ , therefore  $T$  is supra  $\alpha$ -closed.  $\square$

**Lemma 2.1.** *Let  $T$  be supra  $\alpha$ -closed set in the supra space  $X$  which is a domain of a supra homeomorphism function  $h$  and the supra space  $Y$  its co-domain, then  $h(T)$  is a supra  $\alpha$ -closed set in  $Y$ .*

**Proof.** Because of  $T$  is a supra  $\alpha$ -closed subset of  $X$ , so  $cl^\mu(int^\mu(L)) \subseteq T \subseteq L$ , where  $L$  is a supra closed subset of  $X$ , so  $cl^\mu h(int^\mu(L)) = h(cl^\mu(int^\mu(L))) \subseteq h(T) \subseteq h(L)$ , but  $h(L)$  is supra closed subset of  $Y$ , which implies  $h(T)$  is supra  $\alpha$ -closed.  $\square$

**Definition 2.10.** *If  $g(L)$  is supra  $\alpha$ -open (or supra  $\alpha$ -closed) set in  $(Y, \mu_Y)$  for each supra open (supra closed) set  $L$  in the supra space  $(X, \mu_X)$ , hence  $g : (X, \mu_X) \rightarrow (Y, \mu_Y)$  is supra  $\alpha$ -open (supra  $\alpha$ -closed) function.*

**Definition 2.11.** *In Definition 2.10 if  $L$  is supra  $\alpha$ -open (or supra  $\alpha$ -closed) subset of  $(X, \mu_X)$  and its image is supra open (supra closed) subset of  $(Y, \mu_Y)$ , then  $g$  is supra  $\alpha^*$ -open (supra  $\alpha^*$ -closed) function.*

**Definition 2.12.** *In Definition 2.10 whenever  $L$  and  $g(L)$  are supra  $\alpha$ -open (or supra  $\alpha$ -closed) subsets of the supra spaces  $(X, \mu_X)$  and  $(Y, \mu_Y)$  respectively, hence  $g$  is called supra  $\alpha^{**}$ -open (supra  $\alpha^{**}$ -closed) function.*

**Definition 2.13** ([12]). *The function  $h : (X, \mu_X) \rightarrow (Y, \mu_Y)$  is supra  $\alpha$ -continuous, whenever  $h^{-1}(S)$  is supra  $\alpha$ -open subset of  $X$  for any supra open subset  $S$  of  $Y$ .*

**Definition 2.14.** *In Definition 2.13 if  $S$  is supra  $\alpha$ -open subset and its inverse image is supra open (supra  $\alpha$ -open), then  $g$  is supra  $\alpha^*$ -continuous (supra  $\alpha^{**}$ -continuous).*

**Definition 2.15.** *If any supra compact subset in  $(X, \mu_X)$  is supra closed set, so  $X$  is supra  $(\alpha K)$ c-space.*

**Example 2.7.**  $(R, mu_D)$  is supra  $(\alpha K)$ c-space.

**Theorem 2.2.** *Let the supra space  $(X, \mu_X)$  be supra  $(\alpha K)$ c-space and  $h$  from  $(X, \mu_X)$  onto the supra space  $(Y, \mu_Y)$  is injective, supra  $\alpha^*$ -open and supra closed function, hence the space  $Y$  is supra  $(\alpha K)$ c-space.*

**Proof.** Consider  $B$  is a supra  $\alpha$ -compact set in  $Y$ . Also  $\{H_i, i \in I\}$  be a supra  $\alpha$ -open cover for  $h^{-1}(B) \implies h^{-1}(B) \subseteq \cup_{i \in I} H_i \implies h(h^{-1}(B)) \subseteq h(\cup_{i \in I} H_i) \implies B \subseteq \cup_{i \in I} h(H_i)$  where all  $h(H_i), i \in I$  are supra  $\alpha$ -open subsets of  $Y$ , so there is a finite collection from the sets  $h(H_i)$  in which  $B \subseteq \cup_{i=1}^n h(H_i) \implies h^{-1}(B) \subseteq h^{-1}(\cup_{i=1}^n h(H_i)) \implies h^{-1}(B) \subseteq \cup_{i=1}^n h^{-1}(h(H_i)) \implies h^{-1}(B) \subseteq \cup_{i=1}^n H_i \implies h^{-1}(B)$  is supra  $\alpha$ -compact subset of  $X$ , therefore  $h^{-1}(B)$  is a supra  $\alpha$ -closed (because  $X$  is supra  $(\alpha K)$ c-space). So  $h(h^{-1}(B)) = B$  is supra closed subset of  $Y$ , there upon  $B$  is a supra  $\alpha$ -closed, so  $(Y, \mu)$  will be a supra  $(\alpha K)$  c-space.  $\square$

**Corollary 2.1.** *Let the supra space  $(X, \mu_X)$  be supra  $(\alpha K)c$ -space and  $g$  from  $(X, \mu_X)$  onto the supra space  $(Y, \mu_Y)$  is injective, supra open (supra open) and supra  $\alpha^*$ -closed (supra  $\alpha^{**}$ -closed) function, hence the space  $Y$  is supra  $(\alpha K)c$ -space.*

**Proposition 2.4.** *Let the supra space  $(X, \mu_X)$  be supra  $\alpha K(\alpha c)$ -space and  $h$  from  $(X, \mu_X)$  onto the supra space  $(Y, \mu_Y)$  is injective, supra  $\alpha$ -open and supra  $\alpha^{**}$ -closed function, hence the space  $Y$  is supra  $\alpha K(\alpha c)$ -space.*

**Proof.** Consider  $\{H_{<i>i> | i \in I\}$  is a supra  $\alpha$ -open covering for the inverse image of  $B \implies h^{-1}(B) \subseteq \cup_{i \in I} H_i \implies h(h^{-1}(B)) \subseteq h(\cup_{i \in I} H_i) \implies B \subseteq \cup_{i \in I} h(H_i)$ , in which each  $h(H_i)$  is supra  $\alpha$ -open set in  $Y$ , but  $B$  is supra  $\alpha$ -compact, so there is a finite subfamily of the sets  $h(H_i)$  in which  $B \subseteq \cup_{i=1}^n h(H_i) \implies h^{-1}(B) \subseteq \cup_{i=1}^n H_i \implies h^{-1}(B)$  is supra  $\alpha$ -compact subset from the supra space  $X$ , therefore it is a supra  $\alpha$ -closed (since  $X$  is a supra  $\alpha K(\alpha c)$ -space). Hence  $h(h^{-1}(B)) = B$  is supra  $\alpha$ -closed subset from the supra space  $Y$ , so  $Y$  is a supra  $\alpha K(\alpha c)$ -space.  $\square$

**Proposition 2.5.** *When  $h : (X, \mu_X) \rightarrow (Y, \mu_Y)$  is onto supra  $\alpha^*$ -continuous function, where  $X$  is supra  $\alpha$ -compact (or supra  $\alpha$ -Lindelöf) space, hence  $Y$  is also supra  $\alpha$ -compact (supra  $\alpha$ -Lindelöf).*

**Proof.** Consider the supra  $\alpha$ -open cover  $\{C_i | i \in I\}$  to the supra space  $Y$ , so  $Y \subseteq \cup_{i \in I} C_i \implies h^{-1}(Y) \subseteq h^{-1}(\cup_{i \in I} C_i) \implies X \subseteq \cup_{i \in I} h^{-1}(C_i)$  (since  $h$  is onto), where each of  $h^{-1}(C_i), i \in I$  is supra  $\alpha$ -open subset from the supra space  $X$ , hence the collection of  $h^{-1}(C_i), i \in I$  is supra  $\alpha$ -open cover to the supra space  $X$  which is supra  $\alpha$ -compact, so there is a finite sub collection from  $h^{-1}(C_i) | i \in I$  in which  $X \subseteq \cup_{i=1}^n h^{-1}(C_i) \implies h(X) \subseteq h(\cup_{i=1}^n h^{-1}(C_i)) \rightarrow Y \subseteq \cup_{i=1}^n h(h^{-1}(C_i)) \implies Y \subseteq \cup_{i=1}^n C_i$ , which means  $Y$  is supra  $\alpha$ -compact space. The other possibility can be proved by the same way.  $\square$

**Proposition 2.6** ([9]). *A finite supra space, is supra  $\alpha$ -compact space.*

**Proposition 2.7.** *Suppose  $(X, \mu_X)$  is supra  $K(\alpha c)$ -space and  $h : (X, \mu_X) \rightarrow (Y, \mu_Y)$  is supra homeomorphism function, hence  $(Y, \mu_Y)$  is also supra  $K(\alpha c)$ -space.*

**Proof.** Suppose  $S$  is a supra compact subset from the supra space  $(Y, \mu_Y)$ ,  $h^{-1}(S)$  is supra compact sub of  $(X, \mu_X)$ , but  $X$  is supra  $K(\alpha c)$ -space, thus  $h^{-1}(S)$  is supra  $\alpha$ -closed. Now  $h(h^{-1}(S)) = S$  is supra  $\alpha$ -closed subset from  $Y$  ( $h$  is surjective function), and this lead us to the required.  $\square$

**Definition 2.16.** *Whenever any supra Lindelöf subset from the supra space  $(X, \mu_X)$  is supra  $\alpha$ -closed, thus  $X$  is supra  $L(\alpha c)$ -space.*

**Example 2.8.**  $(Z, \mu_{ind})$  is supra  $L(\alpha c)$ -space.

**Remark 2.3.** Any supra  $L_c$ -space is supra  $L(\alpha c)$ -space.  $(Z, \mu_{ind})$  is an example about the converse,

**Remark 2.4.** Suppose the supra space  $X$  is supra  $L(\alpha c)$ -space, hence it will be supra  $K(\alpha c)$ -space, because when  $B$  is supra compact subset from the space  $X$ , it will be supra Lindelöf, Since  $X$  is supra  $L(\alpha c)$ -space,  $B$  will be supra  $\alpha$ -closed set, then  $X$  is supra  $K(\alpha c)$ -space.

**Remark 2.5.** Suppose  $X$  is supra Lindelöf space and  $Y$  is a subspace of  $X$ , so  $Y$  is not necessary supra Lindelöf. But if any sub space of supra Lindelöf (supra  $\alpha$ -Lindelöf) is supra Lindelöf (supra  $\alpha$ -Lindelöf) sub space we called it a supra hereditary Lindelöf ( $\alpha$ -Lindelöf).

**Example 2.9.** The excluded point supra space  $(R, \mu_{EX})$  is supra Lindelöf space but  $(R - \{x_0\}, \mu_{EX})$  is not supra Lindelöf.

### Acknowledgments

We wish to express our sincere thanks to the Mustansiriyah University, College of Science, Department of Mathematics for supporting this research.

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Accepted: April 18, 2020