وزارة التعليم العالي والبحث العلمي
الجامعة المستنصرية
كلية الإدارة و الإقتصاد
قسم الإحصاء

الاستدلال الاحصائي (التقديرات)

- Definition of estimation.
- Graphical estimation.
- Method of point estimation.
- Unbiasedness.
- Mean squared error.
- Consistency.
- Sufficient statistics.
- Rao–black well theorem.
- Crammer Rao inequality.
- Introduction and definition.
- Confidence interval for mean.
- Confidence interval for differ.
- Confidence interval for variance.
- Confidence interval for ratio.
- Applications.

استاذ المادة (1) ، (2)
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1 - البروفايل الخاص بالأستاذ:
https://uomustansiriyah.edu.iq/e-learn/profile.php?id=3290

2- مدخل في الاستدلال الاحصائي ، د. عبد المجيد حمزة الناصر

بعض التوزيعات الخاصة: (Some Special Distribution)

اولاً: التوزيعات المتقطعة (Discrete Distribution)

توزيع برنولي: Bernoulli Distribution

$$f(X;p)=\begin{cases} p^x(1-p)^{1-x} & ,x=0,1 \\ 0 & ,otherwise \end{cases}$$

$$E(x)=p \quad ,var(x)=p(1-p)$$

توزيع ذي الحدين: Binomial Distribution

$$f(X;n,p)=\begin{cases} \binom{n}{x} p^x(1-p)^{n-x} & ,x=0,1,\dots,n \\ 0 & ,otherwise \end{cases}$$

$$E(x)=np \quad ,var(x)=np(1-p)$$

توزيع بواسون: Poisson Distribution

$$f(X,\lambda)=\begin{cases} \frac{e^{-\lambda}(\lambda)^x}{x!} & ,x=0,1,\dots \\ 0 & ,otherwise \end{cases}$$

$$E(x)=\lambda \quad ,var(x)=\lambda$$

توزيع ذي الحدين العكسي: Negative Binomial Distribution

$$f(X;r,p)=\begin{cases} \binom{x+r-1}{x} p^r(1-p)^x & ,x=0,1,\dots \\ 0 & ,otherwise \end{cases}$$

$$E(x)=\frac{r(1-p)}{p} \quad ,var(x)=\frac{r(1-p)}{p^2}$$

التوزيع الهندسي: Geometric Distribution

$$f(X;p)=\begin{cases} p(1-p)^x & ,x=0,1,\dots \\ 0 & ,otherwise \end{cases}$$

$$E(x)=\frac{(1-p)}{p} \quad ,var(x)=\frac{(1-p)}{p^2}$$

ثانياً: التوزيعات المستمرة: (Continous Distribution)

التوزيع المنتظم: Uniform Distribution

$$f(X;a,b)=\begin{cases}\dfrac{1}{b-a} & ,a<x<b\\ 0 & ,\text{otherwise}\end{cases}$$

$$E(x)=\frac{a+b}{2} \qquad ,var(x)=\frac{(b-a)^2}{12}$$

التوزيع الطبيعي: Normal Distribution

$$f(X;\mu,\sigma^2)=\begin{cases}\dfrac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(x-\mu)^2}{2\sigma^2}} & ,-\infty<x<\infty\\ 0 & ,\text{otherwise}\end{cases}$$

$$E(x)=\mu \qquad ,var(x)=\sigma^2$$

توزيع كاما: Gamma Distribution

$$f(X;\alpha,\beta)=\begin{cases}\dfrac{1}{\Gamma\alpha\,\beta^\alpha}\,x^{\alpha-1}\,e^{-\frac{x}{\beta}} & ,x>0\\ 0 & ,\text{otherwise}\end{cases}$$

$$E(x)=\alpha\beta \qquad ,var(x)=\alpha\beta^2$$

توزيع كاما العكسي: Negative Gamma Distribution

$$f(X;\alpha,\beta)=\begin{cases}\dfrac{\beta^\alpha}{\Gamma\alpha}\,x^{\alpha-1}\,e^{-\beta x} & ,x>0\\ 0 & ,\text{otherwise}\end{cases}$$

$$E(x)=\frac{\alpha}{\beta} \qquad ,var(x)=\frac{\alpha}{\beta^2}$$

التوزيع الاسي: Exponential Distribution

$$f(X;\alpha,\beta)=\begin{cases}\dfrac{1}{\beta}\,e^{-\frac{x}{\beta}} & ,x>0\\ 0 & ,\text{otherwise}\end{cases}$$

$$E(x)=\beta \qquad ,var(x)=\beta^2$$

التوزيع الاسي العكسي: Negative Exponential Distribution

$$f(X;\alpha,\beta)=\begin{cases}\beta\,e^{-\beta x} & ,x>0\\ 0 & ,\text{otherwise}\end{cases}$$

$$E(x)=\frac{1}{\beta} \qquad ,var(x)=\frac{1}{\beta^2}$$

توزيع بيتا: Beta Distribution

$$f(X;a,b)=\begin{cases}\dfrac{1}{\beta(a,b)}\,x^{a-1}\,(1-x)^{b-1} & ,0<x<1\\ 0 & ,\text{otherwise}\end{cases}$$

$$E(x)=\frac{a}{a+b} \qquad ,var(x)=\frac{ab}{(a+b)^2(a+b+1)}$$

(Method of Estimation) :طرائق التقدير

(Method of Moments):اولاً: طريقة العزوم

Let $X_1, X_2, \dots, X_n$ be a r.s drawn from a population which has a p.d.f $f(x;\theta)$, $\theta \in \Omega$ where the parameter $\theta = \{\theta_1, \theta_2, \dots, \theta_k\}$ is unknown. Our object is to estimate the parameter θ. The concept of the method of moments depends on the idea of estimating the population moments by the Sample moments. Therefore, the procedure of estimate the parameter for this method is at following:

1) Find the population moments
   $\boldsymbol{\mu_r = E(X^r)}$
2) Find the Sample moments
   $\boldsymbol{\hat{\mu}_r = \frac{\sum X_i^r}{n}}$
3) the population moments = the Sample moments
   $\boldsymbol{\mu_r = \hat{\mu}_r}$

The solution of Step (3) (these system of equation) will be the estimation of the estimation of the parameter $\theta = \{\theta_1, \theta_2, \dots, \theta_k\}$.

(Method of Maximum Likelihood): ثانياً: طريقة الامكان الاعظم

The another method for estimating the Parameter θ, which called Maximum Likelihood estimation method.
The concept of this method depend on Likelihood function which can be defined as follows:
Let $X_1, X_2, \dots, X_n$ be a r.s drawn from a population which has a p.d.f $f(x;\theta)$ $\theta \in \Omega$
Then the Likelihood function $L(\theta; X_1, X_2, \dots, X_n)$ is a joint probability density function the random sample $X_1, X_2, \dots, X_n$.

$$L(\theta; X_1, X_2, \dots, X_n) = f(X_1, X_2, \dots, X_n; \theta)$$

$$= f(X_1;\theta) * f(X_2;\theta) * \dots * f(X_n;\theta) = \prod_{i=1}^{n} f(X_i;\theta)$$

In this method we choose an estimate $\boldsymbol{\hat{\theta}}$ (value of the parameter θ) , That maximizes the $L(\theta)$, that is

$$L(\boldsymbol{\hat{\theta}}; X_1, X_2, \dots, X_n) = \max\{L(\theta; X_1, X_2, \dots, X_n), \theta \in \Omega\}$$

Then $\hat{\theta}$ is called the MLE of θ.
To finding out the MLE to the unknown parameter θ , it will be the solution of the following equation:

$$\frac{\partial L(\theta)}{\partial \theta} = 0 \qquad \text{and} \qquad \frac{\partial^2 L(\theta)}{\partial \theta^2} < 0$$

It was notice that is easier to us logarithm of the likelihood function. Furthermore, $Ln(L(\theta))$ or $L(\theta)$ have their maximum at the same value of θ.

Example (1): Let $X_1, X_2, \dots, X_n$ be a r.s from exponential density

$f(x,\theta) = \frac{1}{\theta} e^{-X/\theta}$, $I(0,\infty)^{(X)}$, Zero otherwise , use the method of moment to estimate θ.

Solution: $X \sim \exp(\theta) \rightarrow E(X) = \theta$ and $var(X) = \theta^2$

1) $\mu_r = E(X^r) \rightarrow \quad \mu_1 = E(X) = \theta$

2) $\hat{\mu}_r = \frac{\sum X_i{}^r}{n} \rightarrow \quad \hat{\mu}_1 = \frac{\sum X_i}{n} = \bar{X}$

3) $\mu_1 = \hat{\mu}_1 \rightarrow \boxed{\hat{\theta} = \bar{X}}$

$\therefore \bar{X}$ is moment estimator for θ

Example (2): Let $X_1, X_2, \dots, X_n$ be a r.s from Binomial distribution.

Estimate the parameter P by using method of moment.

Solution: $X \sim bin(n, P) \rightarrow E(X) = nP$ and $var(X) = nP(1-P)$

$f(x,\theta) = \binom{n}{x} P^x (1-P)^{n-x}$, $x = 0,1,\dots,n$, Zero otherwise

1) $\mu_r = E(X^r) \rightarrow \quad \mu_1 = E(X) = nP$

2) $\hat{\mu}_r = \frac{\sum X_i{}^r}{n} \rightarrow \quad \hat{\mu}_1 = \frac{\sum X_i}{n} = \bar{X}$

3) $\mu_1 = \hat{\mu}_1 \rightarrow \boxed{\hat{P} = \frac{\bar{X}}{n}}$

$\therefore \frac{\bar{X}}{n}$ is moment estimator for P

Example (3): Let $X_1, X_2, \dots, X_n$ be a r.s from Normal distribution.

Estimate the two parameters μ and σ^2 by using method of moment.

Solution: $X \sim N(\mu, \sigma^2) \rightarrow E(X) = \mu$ and $var(X) = E(X^2) - (E(X))^2 = \sigma^2$

$f(x,\mu,\sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(X-\mu)^2}{\sigma^2}}$, $-\infty \leq x \leq \infty$, Zero otherwise

1) $\mu_r = E(X^r) \rightarrow \quad \mu_1 = E(X) = \mu$

2) $\hat{\mu}_r = \frac{\sum X_i{}^r}{n} \rightarrow \quad \hat{\mu}_1 = \frac{\sum X_i}{n} = \bar{X}$

3) $\mu_1 = \hat{\mu}_1 \rightarrow \boxed{\hat{\mu} = \bar{X}}$

4) $\mu_r = E(X^r) \rightarrow \quad \mu_2 = E(X^2) = var(X) + (E(X))^2 = \sigma^2 + \mu^2$

5) $\hat{\mu}_r = \frac{\sum X_i{}^r}{n} \rightarrow \quad \hat{\mu}_2 = \frac{\sum X_i{}^2}{n}$

6) $\mu_2 = \hat{\mu}_2 \rightarrow \hat{\sigma}^2 + \hat{\mu}^2 = \frac{\sum X_i^2}{n} \rightarrow \hat{\sigma}^2 = \frac{\sum X_i^2}{n} - \bar{X}^2$

$\boxed{\hat{\sigma}^2 = S^2 = \frac{\sum(X_i-\bar{X})^2}{n}}$, where $\sum(X_i - \bar{X})^2 = \sum X_i^2 - n\bar{X}^2$

$\therefore$ $\bar{X}$ and S^2 are moment estimators for μ and σ^2

Example (4): Let $X_1, X_2, \dots, X_n$ be a r.s from Gamma distribution.

Estimate the two parameters α and β by using method of moment.

Solution: $X \sim gamma(\alpha, \beta) \rightarrow E(X) = \alpha\beta$ and $var(X) = E(X^2) - \left(E(X)\right)^2 = \alpha\beta^2$

$f(x, \mu, \sigma^2) = \frac{x^{\alpha-1}}{\Gamma\alpha\, \beta^{\alpha}} e^{-\frac{1}{\beta}x}$, $x \geq 0$, Zero otherwise

1) $\mu_r = E(X^r) \rightarrow \qquad \mu_1 = E(X) = \alpha\beta$

2) $\hat{\mu}_r = \frac{\sum X_i^r}{n} \rightarrow \qquad \hat{\mu}_1 = \frac{\sum X_i}{n} = \bar{X}$

3) $\mu_1 = \hat{\mu}_1 \rightarrow \boxed{\hat{\alpha}\hat{\beta} = \bar{X} \rightarrow \hat{\alpha} = \frac{\bar{X}}{\hat{\beta}}}$

4) $\mu_r = E(X^r) \rightarrow \qquad \mu_2 = E(X^2) = var(X) + \left(E(X)\right)^2 = \alpha\beta^2 + (\alpha\beta)^2$

5) $\hat{\mu}_r = \frac{\sum X_i^r}{n} \rightarrow \qquad \hat{\mu}_2 = \frac{\sum X_i^2}{n}$

6) $\mu_2 = \hat{\mu}_2 \rightarrow \hat{\alpha}\hat{\beta}^2 + \left(\hat{\alpha}\hat{\beta}\right)^2 = \frac{\sum X_i^2}{n} \rightarrow \frac{\bar{X}}{\hat{\beta}}\hat{\beta}^2 + (\bar{X})^2 = \frac{\sum X_i^2}{n}$

$\bar{X}\hat{\beta} = S^2 \rightarrow \boxed{\hat{\beta} = \frac{S^2}{\bar{X}}} \rightarrow \because \hat{\alpha} = \frac{\bar{X}}{\hat{\beta}} = \bar{X} * \frac{\bar{X}}{S^2} \rightarrow \boxed{\hat{\alpha} = \frac{\bar{X}^2}{S^2}}$

$\boxed{S^2 = \frac{\sum(X_i-\bar{X})^2}{n}}$, where $\sum(X_i - \bar{X})^2 = \sum X_i^2 - n\bar{X}^2$

$\therefore \frac{\bar{X}^2}{S^2}$ and $\frac{S^2}{\bar{X}}$ are moment estimators for α and β