Statistics Department

For Master student /Lesson (6)

Analysis of Experimental Design

- chapter Five

(Latin Square Design)

-Chapter Six

(Graeco Latin Square Design)

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Chapter Five ch. 5

LATIN SQUARE DESIGNS

A Latin square design involves $(p \geq 2)$ treatment and the experimental units are arranged according to two blocking factors which we label as rows and columns.
There are (p) rows and (p) columns each treatment appears exactly once in each row and each column.

As an illustration consider a Latin square design with $(p=3)$, one possible such LSD follows.

A	B	C
B	C	A
C	A	B

we note that other lation squares are possible by assiging the treatments in a different order Ideally in practiec one should choose the latin square at random. we also note that only some of the row. column and treatment combination are present in a latin square. If all such combination were present in a Latin square then (p^3) observations should be required. A Latin square requires only (p^2) observation.

To facilitate the Analysis of data from an LSD we introduce a model to explain the variation in the results. we let $Y_{ij(k)}$ represent the response to treatment i-th row and j-th column. The subscript (k) is placed in parentheses to indicate that (k) depends on (i and j), that is only one (k) value corresponds to a particular (i,j) combination, our model is then

$$Y_{ij(k)} = \mu + \rho_i + \gamma_j + \tau_k + \epsilon_{ij(k)}$$

$$i, j, k = 1, 2, 3, ---, p$$

where:

μ: is the over all average response to all treatments.

ρ_i : is the effect of ith row.

γ_j: is the effect of j-th column.

τ_k: is the effect of k-th treatment.

$\epsilon_{ij(k)}$: is the random error associated with the k-th treatment applied to ith row and jth coulmn with

$E(\epsilon_{ij(k)}) = 0$ and $var(\epsilon_{ij(k)}) = \sigma^2$ for every i and j. Assumed that $\epsilon_{ij(k)}$ are uncorrelated.

we note that this Model implies that

$E(y_{ij(k)}) = \mu + \rho_i + \gamma_j + \tau_k$ and

$var(y_{ij(k)}) = \sigma^2$.

All observation have the same variance but the mean can vary according to the row, column and treatment.

- Least squares Results:

we first note that the model for an (LSD) is alinear model and can be written as $y = X\beta + \epsilon$. To illustrate this fact consider the LSD when ($p=3$) with observations described as follows:

$y_{11(1)}$	$y_{12(2)}$	$y_{13(3)}$
$y_{21(2)}$	$y_{22(3)}$	$y_{23(1)}$
$y_{31(3)}$	$y_{32(1)}$	$y_{33(2)}$

The model in matrix form is

$$\underline{Y} = \underline{X}\beta + \underline{\epsilon}$$

$$\begin{bmatrix} y_{11(1)} \\ y_{12(2)} \\ y_{13(3)} \\ y_{21(2)} \\ y_{22(3)} \\ y_{23(1)} \\ y_{31(3)} \\ y_{32(1)} \\ y_{33(2)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mu \\ \rho_1 \\ \rho_2 \\ \rho_3 \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} + \begin{bmatrix} \epsilon_{11(1)} \\ \epsilon_{12(2)} \\ \epsilon_{13(3)} \\ \epsilon_{21(2)} \\ \epsilon_{22(3)} \\ \epsilon_{23(1)} \\ \epsilon_{31(3)} \\ \epsilon_{32(1)} \\ \epsilon_{33(2)} \end{bmatrix}$$

As expected the (x) matrix is not full rank Note that the sum of column 2, 3, 4 equals column one the sum is true for columns 4, 5, 6 and column 7, 8, 9 As a consequence, the rank of (x) for this example is $1+(3-1)+(3-1)+(3-1)=7$ It is easy to generalize and conclude that the rank of x for any LSD will be $1+3(p-1)=3p-2$.

To find the normal equation we need some notation, our sample form for the model is:

$$y_{ij(k)} = m + r_i + c_j + t_k + e_{ij(k)} \qquad , i, j, k = 1, 2, \ldots, p$$

As before we let $y_{i..}$, $y_{.j.}$ and $y_{..k}$ represent the sum of the observation in the ith rows, jth column, and kth treatment.
$y_{...}$ is the sum of all p^2 observation. To denote the corresponding average we use bars over the symbol, with this notations it

can be shown the set of normal equations for the LSD in general is

$$p^2 m + p\sum_{i=1}^{p} r_i + p\sum_{j=1}^{p} c_j + p\sum_{k=1}^{p} t_k = y_{\cdot\cdot}$$

$$pm + pr_i + \sum_{j=1}^{p} c_j + \sum_{k=1}^{p} t_k = y_{i\cdot\cdot} \qquad i=1,2,\dots,p$$

$$pm + \sum_{i=1}^{p} r_i + pc_j + \sum_{k=1}^{p} t_k = y_{\cdot j\cdot} \qquad j=1,2,\dots,p$$

$$pm + \sum_{i=1}^{p} r_i + \sum_{j=1}^{p} c_j + pt_k = y_{\cdot\cdot k} \qquad k=1,2,\dots,p$$

Since the (x) matrix is rank deficient by three we need three side conditions to obtain a unique solution. if we impose the conditions:

$\sum_{i=1}^{p} r_i = \sum_{j=1}^{p} c_j = \sum_{k=1}^{p} t_k = 0$ on the system we obtain the Solution set.

$$m = \bar{y}_{\cdot\cdot\cdot}$$
$$r_i = \bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot\cdot\cdot} \qquad , i=1,2,\dots,p$$
$$c_j = \bar{y}_{\cdot j\cdot} - \bar{y}_{\cdot\cdot\cdot} \qquad , j=1,2,\dots,p$$
$$t_k = \bar{y}_{\cdot\cdot k} - \bar{y}_{\cdot\cdot\cdot} \qquad , k=1,2,\dots,p$$

we can now find a formula for

$SSe = \sum_{i=1}^{p}\sum_{j=1}^{p} (y_{ij(k)} - \hat{y}_{ij(k)})^2$ and since

$\hat{y}_{ij(k)} = m + r_i + c_j + t_k$ then

$= \bar{y}_{i\cdot\cdot} + \bar{y}_{\cdot j\cdot} + \bar{y}_{\cdot\cdot k} - 2\bar{y}_{\cdot\cdot\cdot}$ it follows that

$$SSe = \sum_{i=1}^{p}\sum_{j=1}^{p} (y_{ij(k)} - \bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot j\cdot} - \bar{y}_{\cdot\cdot k} + 2\bar{y}_{\cdot\cdot\cdot})^2$$

The degree of freedom for SSe is
$n - \text{rank}(X) = p^2 - (3p-2) = (p-1)(p-2)$

The error mean square for an LSD is then

$$S^2 = SSe/(p-1)(p-2)$$

To find least squares estimates for quantities, we first need to identify what is estimable.

- Infrence for an LSD :-

To make statistical infrence for data from an LSD we need to added assumption That the $\epsilon_{ij(k)}$'s are independent $N(0, \sigma^2)$ random variables. we first turn our attention to the f-test and ANOVA table for an LSD, To test Ho: No differences in treatment effects we use the principle of conditional error. The hypothesis $H_0: \tau_1 = \tau_2 = --- = \tau_p$ can be written in the form of a general linear hypothesis.

$H_0: L\beta = 0$

Here L is a $(p-1) \times (3p+1)$ matrix such that $L\beta$ contains as its $(p-1)$ rows the linear independent estimable functions $\tau_2 - \tau_1, \tau_3 - \tau_1, ---, \tau_p - \tau_1$.

Imposing the hypothesis $L\beta = 0$ of the full model we obtaine the reduce model.

$$y_{ij(k)} = \mu^* + \rho_i + \gamma_j + \epsilon^*_{ij(k)} \qquad i,j = 1,2,\ldots,p$$

since there is only one k- value for each (i,j) pair, we observe that this reduced model is an RCBD model, using the notation for an LSD we arrive at.

$$SSe^* = \sum_{i=1}^{p} \sum_{j=1}^{p} (y_{ij(k)} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...})^2$$

The treatment sum of Squares

$$sstr = SSe^* - SSe.$$

we know that $\mu + \rho_i + \gamma_j + \tau_k$ is estimable for every (i,j,k) for which an observation is present.

we can now find the BLUE for a treatment contrast $\sum_{k=1}^{p} c_k \tau_k$, $\sum_{k=1}^{p} c_k = 0$

using the Solution for the N.E, the BLUE for the contrast is

$$\sum_{k=1}^{p} c_k (\bar{y}_{..k} - \bar{y}_{...}) = \sum_{k=1}^{p} c_k \bar{y}_{..k}$$

To find the standard error we first find the variance, since observation are independent we obtain.

$$\text{var}\left(\sum_{k=1}^{p} c_k \bar{y}_{..k}\right) = \sum_{k=1}^{p} c_k^2 \,\text{var}(\bar{y}_{..k}) = \sum_{k=1}^{p} c_k^2 \sigma^2/p$$

Replace σ^2 by S^2 and take the Square root to obtain the standard error.

$$S.E\left(\sum_{k=1}^{p} C_k \bar{y}_{..k}\right) = \sqrt{(S^2/p)\left(\sum_{k=1}^{p} C_k^2\right)}$$

where

$$SSe = \sum_{i=1}^{p}\sum_{j=1}^{p}\left(y_{ij(k)} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...}\right)^2$$

we arrive at

$$SStr = p\sum_{k=1}^{p}\left(\bar{y}_{..k} - \bar{y}_{...}\right)^2$$

Hence the statistic used to test

$H_0: \tau_1 = \tau_2 = \_\_\_ = \tau_p$ is

$$F = \frac{SStr/p-1}{S^2} = \frac{MStr}{MSe}$$

we reject Ho if $f \geqslant f_{\alpha, p-1, (p-1)(p-2)}$

Note that this sstr is actually an adjusted sum of Squares, as we shall discover later on an LSD is orthognal. As a consequences the adjustes and unadjusted sum of Squares are identical.

In a similar manner the hypothesis

$H_0: \rho_1 = \rho_2 = \_\_\_ = \rho_p$ can be tested using the Statistic

$$F = \frac{SSro/p-1}{S^2} = \frac{MSro}{MSe}$$

where

$$SSro = p \sum_{i=1}^{p} (\bar{y}_{i..} - \bar{y}_{...})^2 = \frac{\sum_{i=1}^{p} y_{i..}^2}{p} - \frac{y_{...}^2}{p^2}$$

is called the row sum of Squares.

Also the hypothesis $H_0: \gamma_1 = \gamma_2 = \cdots = \gamma_p$ can be tested using the statistics

$$F = \frac{SSco/(p-1)}{S^2} = \frac{MSco}{MSe}$$

where

$$SS\,co = p \sum_{j=1}^{p} (\bar{y}_{.j.} - \bar{y}_{...})^2 = \frac{\sum_{j=1}^{p} y_{.j.}^2}{p} - \frac{y_{...}^2}{p^2}$$

is called the column Sum of Squares.

$$SSe = SSt - SSro - SSco - SStr$$

we now easy to find a confidence interval for any treatment contrast and, in particular, for a pairwise treatment difference.

In a similar way t-test can be obtaind. These results are summarized as follows

function	confidence Interval
$\tau_k - \tau_t$	$\bar{y}_{..k} - \bar{y}_{..t} \mp t_{\alpha/2,(p-1)(p-2)} \sqrt{\frac{2s^2}{p}}$
$\sum_{k=1}^{p} c_k \tau_k$, $\sum_{k=1}^{p} c_k = 0$	$\sum_{k=1}^{p} c_k \bar{y}_{..k} \mp t_{\alpha/2,(p-1)(p-2)} \sqrt{\frac{s^2}{p} \sum_{k=1}^{p} c_k^2}$

Null hypothesis	Test statistics
$H_0: \tau_k - \tau_t = \delta$	$t = (\bar{y}_{..k} - \bar{y}_{..t} - \delta) / \sqrt{\frac{2s^2}{p}}$
$H_0: \sum_{k=1}^{p} c_k \tau_k = \delta$, $\sum_{k=1}^{p} c_k = 0$	$t = \left(\sum_{k=1}^{p} c_k \bar{y}_{..k} - \delta\right) / \sqrt{\frac{s^2}{p} \sum_{k=1}^{p} c_k^2}$

All rejection regions are based upon a t-distribution with $(p-1)(p-2)$ degrees of fredom for multiple comparisons of the treatments we consider the Tukey procedure. Since $\bar{y}_{..1}, \bar{y}_{..2} ---- \bar{y}_{..p}$ are independent normal random variables with variance (σ^2/p), we can use the Tukey procedure it is summarized in $(100(1-\alpha)\%)$ simultaneous confidence Interval for all pairs $(\tau_k - \tau_t)$:

$$(\bar{y}_{..k} - \bar{y}_{..t}) \mp q_{\alpha,p,(p-1)(p-2)} \cdot \sqrt{\frac{s^2}{p}}$$

Multiple comparison tests for $H_0: \tau_k - \tau_t$

Decide $\tau_k \neq \tau_t$ if $|\bar{y}_{..k} - \bar{y}_{..t}| > q_{\alpha,p(p-1)(p-2)} \sqrt{\frac{s^2}{p}}$

Expected Mean Squares :-

Recall from Theorm the formula

$$E(MS) = \sigma^2 + w/q$$

where W is the expression for the Sum of squares when the y's are replaced by their expected values and (q) is the degress of freedom for the mean square. To find $E(MSTR)$ we recall that

$$SSTR = p\sum_{k=1}^{p}(\bar{y}_{..k} - \bar{y}_{...})^2 \quad \text{therefore}$$

$$W = p\sum_{k=1}^{p}\left[E(\bar{y}_{..k}) - E(\bar{y}_{...})\right]^2$$

For a LSD

$$E(\bar{y}_{...}) = \sum_{i=1}^{p}\sum_{j=1}^{p} y_{ij(k)}/p^2 = \mu + \bar{\rho} + \bar{\gamma} + \bar{\tau}$$

where $\bar{\rho} = \sum_{i=1}^{p} \rho_i/p$; $\bar{\gamma} = \sum_{j=1}^{p} \gamma_j/p$ and

$$\bar{\tau} = \sum_{k=1}^{p} \tau_k/p$$

Similary $E(\bar{y}_{..k}) = \mu + \bar{\rho} + \bar{\gamma} + \tau_k$

therefore

$$W = p\sum_{k=1}^{p}(\tau_k - \bar{\tau})^2 \quad \text{we then obtain}$$

$$E(MSTR) = \sigma^2 + p\sum_{k=1}^{p}(\tau_k - \bar{\tau})^2/p-1$$

In a completely analogous manner we can obtain expressions for the expected mean Square for rows and columns.

ANOVA for LSD with E(MS)

S.o.V	d.f	S.S	EMS
Rows	$p-1$	$\sum_{i=1}^{p} y_{i\cdot\cdot}^2/p - \frac{y_{\cdot\cdot\cdot}^2}{p^2}$	$\sigma^2 + p\sum_{i=1}^{p}(\rho_i-\bar{\rho})^2/p-1$
column	$p-1$	$\sum_{j=1}^{p}\frac{y_{\cdot j\cdot}^2}{p} - \frac{y_{\cdot\cdot\cdot}^2}{p^2}$	$\sigma^2 + p\sum_{j=1}^{p}(\gamma_j-\bar{\gamma})^2/p-1$
treatment	$p-1$	$\sum_{k=1}^{p}\frac{y_{\cdot\cdot k}^2}{p} - \frac{y_{\cdot\cdot\cdot}^2}{p^2}$	$\sigma^2 + p\sum_{k=1}^{p}(\tau_k-\bar{\tau})^2/p-1$
Error	$(p-1)(p-2)$	$SSe = SSt - SSro - SSco - SStr$	σ^2
Total	p^2-1	$\sum_{i=1}^{p}\sum_{j=1}^{p} y_{ij(k)}^2 - \frac{y_{\cdot\cdot\cdot}^2}{p^2}$	

- The Restricted LSD Models -

Some authers include the usual side condition as part of the model, we have referred to this as the restricted model. For the (LSD) the restricted model is:

$$y_{ij(k)} = \mu + \rho_i + \gamma_j + \tau_k + \epsilon_{ij(k)} \qquad i,j,k = 1,2,\dots,p$$

$$\sum_{i=1}^{p}\rho_i = 0 \;;\; \sum_{j=1}^{p}\gamma_j = 0 \;;\; \sum_{k=1}^{p}\tau_k = 0$$

As we have noted befor, for the restricted model all parameters are individually estimable, we observe that the BLUE for μ, β_i, γ_j and τ_k are $\bar{y}_{\cdots}$, $\bar{y}_{i\cdot\cdot} - \bar{y}_{\cdots}$, $\bar{y}_{\cdot j\cdot} - \bar{y}_{\cdots}$ and $\bar{y}_{\cdot\cdot k} - \bar{y}_{\cdots}$

In this restricted case the hypothesis of no treatment effect becoms.

$H_0 : \tau_1 = \tau_2 = \cdots = \tau_k$ Also

$$E(MSTR) = \sigma^2 + p \sum_{k=1}^{p} \tau_k^2 / p-1$$

It should note that there is no difference in the usual analysis for an (LSD) bared on the unrestricted model versus the restricted model.

- Design Considerations :-

The Latin Square offers certain advantages:-

- The experimental error can possibly be reduced by incorporating two blocking factors into the design effective blocking results in more powerful tests and tighter confidence intervals.

- The Analysis of the data is fairly simple and can easily be done even without a computer.

- The design is orthogonal
- The treatment means are comparable

- This three factor design only requires p^2 observation rather than the p^3 observations required if all possible row, column, treatment combinations are present.

There are also some disadvantages to the (LSD):

- The number of levels of all three factors must be the same.

- Not all rows column, treatment combinations are present in the design, some of those missing may be important possibilities as they might be the optimal condition.

- The error degress of fredom is small when $(p \leq 4)$

- The model is an additive one, there is no easy way to check for interaction between the factors.

The LSD is often used in drug studies In some problems a subject is given several drugs and we might be concerned about possible order effects of the drugs. An LSD can be used to help (even out) the order effect of the drug.

As an example consider the case where there are four drugs A, B, C, D and each subject will use each drug in some order, if we use $p=4$ where the treatments are the drugs, the rows are the patients and the columns are the order in which the drug is taken, then the resulting structure could be as follows:

order for Drug

subject	A	B	D	C
	C	A	B	D
	D	C	A	B
	B	D	C	A

Notice that the LSD ensures that each drug is used in each order exactly once we should point out however that the LSD dose not allow for represention of all $4! = 24$ different order for the drugs.

Designs that are used to incorporate order effects are often called crossover designs.

We mentioned that when p is small the degrees of fredome for error for a Latin Squre is small one remedy for this problem is to replicate the latian square. For example if $p=2$ a design in which the latin square is replicated three times would appears as follows:

replication(1)

A	B
B	A

replication (2)

A	B
B	A

replication (3)

A	B
B	A

As an illustration consider a study to compare two varieties of wheat (A, B), In a replicate LSD our rows could represent a fertilizer level (Low, high) our columns could represent the pesticide level (none - Some) and the replicate could represent different fields.

An extension of a Latin square to four factors can be accomplished by using a Graeco - Latin Square design, In such a design there are four factor three blocking factors and a treatment factors. Each treatment occurs once in every row, column and Layer. Each layer occurs once in every row and column.

Exercises :-

1- (Replicated Latin Square)

Consider a Latin square design with (p) rows, columns, and treatments. Suppose that this same design is replicated (q) times so that the levels of the rows and columns are the same for each replication. An illustration of this situation with $p=2$ and $q=3$ is given. A model for this replicated LSD is

$$Y_{ij(k)L} = \mu + \rho_i + \gamma_j + \tau_k + W_L + \epsilon_{ij(k)L}$$

where $Y_{ij(k)L}$ is the observation in the ith row, jth column, kth treatment in the Lth replicated Latin square.

a- find the form of the ANOVA table for this replicated (LSD).

b- Explian why the model and ANOVA table would change if the levels of the rows were different in each replication (This could occure if the row factor is subject and different subjects are used for each replication.

Chapter Six

Graeco Latin Squar Design (GLSD)

Two latin squars are orthognal if when they are combined the same pair of symbols occurs no more than once in the composite squars

LSD ①

a_1	a_2	a_3
a_3	a_1	a_2
a_2	a_3	a_1

LSD ②

b_1	b_2	b_3
b_2	b_3	b_1
b_3	b_1	b_2

LSD ③

c_2	c_3	c_1
c_3	c_1	c_2
c_1	c_2	c_3

The combined L.S① with LS② is

a_1b_1	a_2b_2	a_3b_3
a_3b_2	a_1b_3	a_2b_1
a_2b_3	a_3b_1	a_1b_2

Then square ① and Square ② are orthognal this square is called a Graeco Latin Square.

α_1	β_2	β_3	or	A_1	B_2	C_3
β_2	α_3	γ_2		C_2	A_3	B_1
γ_3	γ_1	α_2		B_3	C_1	A_2

The combined square ② with Square ③

b_1c_2	b_2c_3	b_3c_1
b_2c_3	b_3c_1	b_1c_2
b_3c_1	b_1c_2	b_2c_3

Square ② and square ③ are not orthognal not (GLS).

The model is

$$Y_{ijkl} = \mu + R_i + C_j + \tau_k + L_l + e_{ijkl}$$

$i, j, k, l = 1, 2, \dots, r$

S.o.V	d.f
Rows	$p-1$
Columns	$p-1$
treatment	$p-1$
Letter Greace (add. treat)	$p-1$
Error	$(p-1)(p-3)$
Total	p^2-1

Example:- $p=4$

The treatments are A, B, C, D the Greace letters are $\alpha, \beta, \delta, \gamma$

R \ C	1	2	3	4
1	$B\delta$	$C\alpha$	$A\gamma$	$D\beta$
2	$D\alpha$	$A\delta$	$C\beta$	$B\gamma$
3	$A\beta$	$D\gamma$	$B\alpha$	$C\delta$
4	$C\gamma$	$B\beta$	$D\delta$	$A\alpha$

Example: $P=5$

$$\begin{array}{ccccc} A\alpha & B\beta & C\gamma & D\delta & E\epsilon \\ C\epsilon & D\alpha & E\beta & A\gamma & B\delta \\ E\delta & A\epsilon & B\alpha & C\beta & D\gamma \\ B\gamma & C\delta & D\epsilon & E\alpha & A\beta \\ D\beta & E\gamma & A\delta & B\epsilon & C\alpha \end{array} \xrightarrow[\text{numbers}]{\text{as}} \begin{array}{ccccc} A_1 & B_2 & C_3 & D_4 & E_5 \\ C_5 & D_1 & E_2 & A_3 & B_4 \\ E_4 & A_5 & B_1 & C_2 & D_3 \\ B_3 & C_4 & D_5 & E_1 & A_2 \\ D_2 & E_3 & A_4 & B_5 & C_1 \end{array}$$

- Another kinds of Design:

A reasrch that test (5) **effects** but must satisfy the conditions of L.S.D or G.L.S.D

$P=4$, effects $=5$

$$\begin{array}{cccc} A\alpha a & B\beta b & C\gamma c & D\delta d \\ D\gamma b & C\delta a & B\alpha d & A\beta c \\ C\beta d & D\alpha c & A\delta b & B\gamma a \\ B\delta c & A\gamma d & D\beta a & C\alpha b \end{array}$$

S.o.V	d.f
R	4-1
C	4-1
t_1	4-1
t_2	4-1
t_3	4-1
Error	0
Total	15

we cannot make this Design only if $P=5$ and more than (5) , Sometimes we can partition one of the components to become Error tearm.

Remarke:- Cochran said if the amount $(\frac{P}{4})$ and the rest is (2) then the Design cannot be made or planned but if $(\frac{10}{4})$ and the rest (2) then we find this Design only ~~too~~ and not find $P=6$.

Example:- If we have $P=10$ then $\frac{10}{4}=2$ and the rest (2), the Design is:

C \ R	1	2	3	4	5	6	7	8	9	10
1	3_4	10_1	1_5	6_9	9_6	5_{10}	7_8	4_3	2_7	8_2
2	7_6	6_5	4_{10}	1_3	3_2	10_9	2_1	8_7	5_4	9_8
3	9_7	5_2	3_8	10_6	6_{10}	7_1	4_5	1_9	8_3	2_4
4	5_5	4_4	9_9	2_2	1_1	8_8	3_3	6_6	10_{10}	7_7
5	4_1	2_8	6_4	7_{10}	10_7	9_2	8_9	3_5	1_6	5_3
6	8_{10}	7_9	5_7	3_1	2_3	4_6	1_2	9_4	6_8	10_5
7	1_8	8_6	7_3	9_5	5_9	3_7	10_4	2_{10}	4_2	6_1
8	6_2	9_3	8_1	4_7	7_4	2_5	5_6	10_8	3_9	1_{10}
9	2_9	1_7	10_2	8_4	4_8	6_3	9_{10}	5_1	7_5	3_6
10	10_3	3_{10}	2_6	5_8	8_5	1_4	6_7	7_2	9_1	4_9

مثلاً

row :	Age	تغير العمر	= R
column :	weight	تغير الوزن	= C
treat. :	medicen	تغير الدواء	= t
latterGrage :	Doctor	تغير الطبيب	= دكورلاتيني

Write a paper about GLSD find the model , normal equation, ANOVA table , test hypothesis confidence intravels, Reduce model, EMSTR.

Thanks for lessening

Please Read Carefully and write the answer of above questions