H.W

① Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$.

show that:

ⓐ $$\rho_{12\cdot 3} = \frac{\rho_{12} - \rho_{13}\rho_{23}}{\sqrt{(1-\rho_{13}^2)(1-\rho_{23}^2)}}$$

ⓑ $$\rho_{13\cdot 2} = \frac{\rho_{13} - \rho_{12}\rho_{32}}{\sqrt{(1-\rho_{12}^2)(1-\rho_{32}^2)}}$$

ⓒ $$R^2_{1\cdot 23} = 1 - \frac{\Sigma_{11\cdot 2}}{\sigma_{11}}$$

ⓓ $E(X_1 \mid X_2, X_3)$

② Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$ with $\underline{\mu} = \begin{pmatrix} 2 \\ 0 \\ - \end{pmatrix}$

and $\Sigma = \begin{pmatrix} 2 & 0 & 1 \\ & 2 & -1 \\ & & 3 \end{pmatrix}$

Find ⓐ $\rho_{13\cdot 2}$ and $\rho_{32\cdot 1}$

ⓑ $R^2_{1\cdot 23}$ and $R^2_{3\cdot 12}$

H.w

① Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$.

show that:

ⓐ $$\rho_{12.3} = \frac{\rho_{12} - \rho_{13}\rho_{23}}{\sqrt{(1-\rho_{13}^2)(1-\rho_{23}^2)}}$$

ⓑ $$\rho_{13.2} = \frac{\rho_{13} - \rho_{12}\rho_{32}}{\sqrt{(1-\rho_{12}^2)(1-\rho_{32}^2)}}$$

ⓒ $$R^2_{1.23} = 1 - \frac{\Sigma_{11.2}}{\sigma_{11}}$$

ⓓ $E(X_1 / X_2, X_3)$

② Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$ with $\underline{\mu} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}$

and $\Sigma = \begin{pmatrix} 2 & 0 & 1 \\ & 2 & -1 \\ & & 3 \end{pmatrix}$

Find ⓐ $\rho_{13.2}$ and $\rho_{32.1}$

ⓑ $R^2_{1.23}$ and $R^2_{3.12}$

Problem. N. 1

Q1: Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$ with $\underline{\mu} = \underline{0}$ and $\Sigma = \begin{pmatrix} 1 & 2 & -1 \\ & 6 & 0 \\ & & 4 \end{pmatrix}$. Set $Y_1 = X_1 + X_3$, $Y_2 = 2X_1 - X_2$ and $Y_3 = 2X_3 - X_2$

Find

ⓐ The distribution of $\underline{Y}' = (Y_1 \ Y_2 \ Y_3)$

ⓑ The correlation matrix of $\underline{Y}$.

ⓒ The conditional distribution of Y_3 given $Y_1 = 0$.

Q2: Let $\underline{X} = (X_1 \ X_2)'$ has the characteristic function:

$$\phi_{\underline{X}}(t) = \exp\left[it_1 + 2it_2 - \frac{1}{2}t_1^2 + 2t_1t_2 - 6t_2^2\right]$$

Determine the distribution of $\underline{X}$.

Q3: Let $\underline{X} \sim N_2\left[\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}\right]$

Find ⓐ The characteristic function of $\underline{X}' = (X_1 \ X_2)$

ⓑ Let $Y = 3X_1 + 4X_2$, find $\phi_y(t)$

ⓒ Let $Y_1 = X_1 + 2X_2$ and $Y_2 = 2X_1 + X_2$

Find $\Phi_{\underline{Y}}(\underline{t})$, where $\underline{Y} = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$.

Q4: If you have the following moment generating function of $\underline{X}$

$$M_{\underline{X}}(\underline{t}) = \exp(5t_1 + 4t_2 + 11t_1^2 + 8t_2^2 + 17t_1t_2)$$

Find the distribution of $\underline{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$

Q5: Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$ with pdf is

$f(\underline{x}) = K\exp\{-Q/2\}$, where

$$Q = x_1^2 + 2x_2^2 + \frac{1}{3}x_3^2 + 2x_1x_2 + 2x_2 - \frac{4}{3}x_3 + \frac{7}{4}$$

Find ⓐ The mean vector $\underline{\mu}$ and var-cov matrix Σ

ⓑ The value of the normalized constant K.

ⓒ The distribution of $Y_1 = X_1 - X_3$ and $Y_2 = X_2$.

Q6: Let $\underline{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right]$

Derive the density function of $\underline{X} = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$

Q7: If $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$ show that

$E(\underline{X} - \underline{\mu})' \Sigma^{-1} (\underline{X} - \underline{\mu}) = p$.

ex: let $\underset{\sim}{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \sim N_3(\underset{\sim}{\mu}, \Sigma)$

find (1) $\rho_{1.23}$ (2) β (regression coefficient.)

if (a) $\underset{\sim}{x_1} = x_1$ (b) find $\rho_{2.13}$ (c) $\rho_{3.12}$.

(1) $\Sigma = \left(\begin{array}{c|cc} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \hline & \sigma_2^2 & \sigma_{23} \\ & & \sigma_3^2 \end{array}\right)$ $\rho_{1.23}$

[illegible]

$$\rho_{1.23} = \sqrt{\frac{\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}}{\sigma_1^2}}$$

$$\Sigma_{22} = \begin{pmatrix} \sigma_2^2 & \sigma_{12} \\ \sigma_{21} & \sigma_3^2 \end{pmatrix}$$

$$\Sigma_{22}^{-1} = \begin{bmatrix} \sigma_2^2 & \sigma_{12} \\ & \sigma_3^2 \end{bmatrix}^{-1}$$

$$\Sigma_{22}^{-1} = \frac{1}{\sigma_2^2\sigma_3^2(1-\rho_{23}^2)} \begin{pmatrix} \sigma_3^2 & -\rho_{23}\sigma_2\sigma_3 \\ & \sigma_2^2 \end{pmatrix}$$

$$= \frac{1}{(1-\rho_{23}^2)} \begin{pmatrix} \frac{1}{\sigma_2^2} & -\frac{\rho_{23}}{\sigma_2\sigma_3} \\ & \frac{1}{\sigma_3^2} \end{pmatrix}$$

$$\sigma_{12} = \rho_{12}\sigma_1\sigma_2$$
$$\sigma_{13} = \rho_{13}\sigma_1\sigma_3$$

then

$$\rho_{1.23}^2 = \frac{\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}}{\sigma_1^2} = \frac{1}{(1-\rho_{23}^2)\sigma_1^2} \begin{pmatrix} \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3 \end{pmatrix} \begin{pmatrix} \frac{1}{\sigma_2^2} & \frac{-\rho_{23}}{\sigma_2\sigma_3} \\ & \frac{1}{\sigma_3^2} \end{pmatrix} \begin{pmatrix} \rho \dots \end{pmatrix}$$

$$= \frac{1}{\sigma_1^2(1-\rho_{23}^2)}\left(\rho_{12}\frac{\sigma_1}{\sigma_2} - \rho_{13}\rho_{23}\frac{\sigma_1}{\sigma_2} - \rho_{12}\rho_{23}\frac{\sigma_1}{\sigma_3} + \rho_{13}\frac{\sigma_1}{\sigma_3}\right)\begin{pmatrix}\rho_{12}\sigma_1\sigma_2\\ \rho_{13}\sigma_1\sigma_3\end{pmatrix}$$

$$= \frac{1}{\sigma_1^2(1-\rho_{23}^2)}\left(\rho_{12}^2\sigma_1^2 - \rho_{13}\rho_{23}\rho_{12}\sigma_1^2 - \rho_{12}\rho_{23}\rho_{13}\frac{\sigma_1^2\sigma_3}{\sigma_3} + \rho_{13}^2\sigma_1^2\right)$$

$$= \frac{1}{\sigma_1^2(1-\rho_{23}^2)}\left(\rho_{12}^2\sigma_1^2 - \rho_{13}\rho_{23}\rho_{12}\sigma_1^2 - \rho_{12}\rho_{23}\rho_{13}\sigma_1^2 + \rho_{13}^2\sigma_1^2\right)$$

$$= \frac{1}{\sigma_1^2(1-\rho_{23}^2)}\left(\rho_{12}^2\sigma_1^2 - 2\rho_{12}\rho_{23}\rho_{13}\sigma_1^2 + \rho_{13}^2\sigma_1^2\right)$$

$$= \frac{1}{\cancel{\sigma_1^2}(1-\rho_{23}^2)}\cancel{\sigma_1^2}\left(\rho_{12}^2 - 2\rho_{12}\rho_{23}\rho_{13} + \rho_{13}^2\right)$$

$$= \frac{1}{(1-\rho_{23}^2)}(\rho_{12}-\rho_{13})^2$$

$$\boxed{\rho_{1.23}^2 = \frac{(\rho_{12}-\rho_{13})^2}{(1-\rho_{23}^2)}} \quad \text{or} \quad \boxed{\rho_{1.23}^2 = \frac{\rho_{12}^2 - 2\rho_{12}\rho_{23}\rho_{13} + \rho_{13}^2}{(1-\rho_{23}^2)}}$$

$$\Rightarrow \quad \rho_{1.23} = \sqrt{\frac{(\rho_{12}-\rho_{13})^2}{(1-\rho_{23}^2)}}$$

(2) $B = \Sigma_{22}^{-1}\Sigma_{21}$ regression coefficient.

$$= \begin{pmatrix} \sigma_2^2 & \sigma_{23} \\ \sigma_{32} & \sigma_3^2 \end{pmatrix}^{-1} \begin{pmatrix} \sigma_{12} \\ \sigma_{13} \end{pmatrix}$$

$$= \frac{1}{(1-\rho_{23}^2)} \begin{pmatrix} \frac{1}{\sigma_2^2} & \frac{-\rho_{23}}{\sigma_2\sigma_3} \\ \frac{-\rho_{23}}{\sigma_2\sigma_3} & \frac{1}{\sigma_3^2} \end{pmatrix} \begin{pmatrix} \sigma_{12} \\ \sigma_{13} \end{pmatrix}$$

$$= \frac{1}{(1-\rho_{23}^2)} \begin{pmatrix} \rho_{12}\frac{\sigma_1}{\sigma_2} - \rho_{23}\rho_{13}\frac{\sigma_1}{\sigma_2} \\ -\rho_{23}\rho_{12}\frac{\sigma_1}{\sigma_3} + \rho_{13}\frac{\sigma_1}{\sigma_3} \end{pmatrix}$$

$\sigma_{12} = \rho_{12}\sigma_1\sigma_2$
$\sigma_{13} = \rho_{13}\sigma_1\sigma_3$

$$= \frac{1}{(1-\rho_{23}^2)} \begin{pmatrix} \frac{\sigma_1\rho_{12} - \sigma_1\rho_{23}\rho_{13}}{\sigma_2} \\ \frac{\sigma_1(-\rho_{23}\rho_{12} + \rho_{13})}{\sigma_3} \end{pmatrix}$$

$$\underline{\underline{B}} = \begin{pmatrix} \frac{\sigma_1(\rho_{12} - \rho_{23}\rho_{13})}{(1-\rho_{23}^2)} \\ \frac{\sigma_1\rho_{13}(1-\rho_{23})}{\sigma_3(1-\rho_{23}^2)} \end{pmatrix}$$

ex: finding the Σ matrix and μ vector from Quadratic forms. if $x \sim N_p(\mu, \Sigma)$ then the p.d.f. of $\underline{x}$ is

$$f(\underline{x}) = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} \exp\left\{-\frac{1}{2} (\underline{x}-\underline{\mu})' \Sigma^{-1} (\underline{x}-\underline{\mu})\right\}$$

$$= \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}} \exp\left\{-\frac{1}{2} Q\right\} = \boxed{K e^{-\frac{1}{2}Q}}$$

$\ni$ $K = \frac{1}{(2\pi)^{p/2} |\Sigma|^{1/2}}$, Q is the Q.f. with symmetric Σ

(i) To find $\underline{\mu}$

$$Q = (\underline{x}-\underline{\mu})' \Sigma^{-1} (\underline{x}-\underline{\mu})$$

$$Q = \underline{x}' \Sigma^{-1} \underline{x} - 2x' \Sigma^{-1} \mu + \mu' \Sigma^{-1} \mu$$

$$\frac{\partial Q}{\partial x} = 2 \Sigma^{-1} x - 2 \Sigma^{-1} \mu$$

$$0 = 2 \Sigma^{-1} x - 2 \Sigma^{-1} \hat{\mu}$$

$$\hat{\mu} = \tilde{x}$$

That is $\begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_p \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_p \end{bmatrix}$

or

let $M = \Sigma$

$Q = (\underline{x}-\underline{\mu})' \Sigma^{-1} (\underline{x}-\underline{\mu})$

$Q = (\underline{x}-\underline{\mu})' M (\underline{x}-\underline{\mu})$

$Q = \underline{x}' M \underline{x} - 2\underline{x}' M \underline{\mu} + \underline{\mu}' M \underline{\mu}$

$\frac{\partial Q}{\partial \underline{x}} = 2M\underline{x} - 2M\hat{\mu} = 0$

$M x = M \hat{\mu}$

$\underline{x} = \hat{\mu}$

since $\underline{x}' M \underline{x}$ is the only term that involves second degree terms in $\underline{x}$. Thus from Q we selected only the second degree terms and evaluate the matrix M.

(2) To find the matrix Σ from the constant of the Quadratic (of power 2). in the diagonal elements of Σ^{-1} matrix in the parition and the half constant of the x_i, x_j's is the elements of the diagonal in the ith row jth Column

$$\frac{\partial \ln L}{\partial \mu} = -\frac{1}{2} \sum_{j=1}^{N} (-2) \Sigma^{-1} (\underline{x}_j - \mu)$$

$$0 = -\frac{1}{\cancel{2}} \sum_{j=1}^{N} (-\cancel{2}) \Sigma^{-1} (\underline{x}_j - \hat{\mu})$$

$$\sum_{j=1}^{N} \Sigma^{-1} (\underline{x}_j - \hat{\mu}) = 0$$

$$\Sigma^{-1} \sum_{j=1}^{N} (\underline{x}_j - \hat{\mu}) = 0$$

$$\sum_{j=1}^{N} (\underline{x}_j - \hat{\mu}) = 0$$

$$\sum_{j=1}^{N} \underline{x}_j - N\hat{\mu} = 0$$

$$\sum_{j=1}^{N} x_j = N\hat{\mu}$$

$$\hat{\mu} = \frac{\sum_{j=1}^{N} x_j}{N}$$

$$\frac{\partial \ln L}{\partial \Sigma} = -\frac{N}{2}\frac{\text{Adj}\,\Sigma}{|\Sigma|} - \frac{1}{2}\sum_{j=1}^{N}(-1)\Sigma^{-1}(\underline{x}_j-\mu)(\underline{x}_j-\mu)'\Sigma^{-1}$$

<u>Note</u>

(1) $$\frac{\partial \ln|\Sigma|}{\partial \Sigma} = \frac{\text{Adj}(\Sigma)}{|\Sigma|} = \Sigma^{-1}$$

(2) $$\frac{\partial}{\partial \Sigma}(\underline{x}-\underline{\mu})'\Sigma^{-1}(\underline{x}-\underline{\mu}) = -\Sigma^{-1}(x-\mu)(x-\mu)'\Sigma^{-1}$$

(3) $$\frac{\partial|\Sigma|}{\partial \Sigma} = \text{Adj}\,\Sigma$$

$$\frac{\partial \ln L}{\partial \Sigma} \doteq 0$$

$$0 = \frac{-N}{2}\frac{\text{Adj}\,\hat{\Sigma}}{|\hat{\Sigma}|} + \frac{1}{2}\sum_{j=1}^{N}\hat{\Sigma}^{-1}(x_j-\hat{\mu})(x_j-\hat{\mu})'\hat{\Sigma}^{-1}$$

$$0 = \frac{-N}{2}\hat{\Sigma}^{-1} + \frac{1}{2}\sum_{j=1}^{N}\hat{\Sigma}^{-1}(x_j-\hat{\mu})(x_j-\hat{\mu})'\hat{\Sigma}^{-1}$$

$$0 = \frac{1}{2}\left(-N + \sum_{j=1}^{N}\hat{\Sigma}^{-1}(\underline{x}_j-\hat{\underline{\mu}})(\underline{x}_j-\hat{\underline{\mu}})'\right)\hat{\Sigma}^{-1}$$

$$0 = -N + \sum_{j=1}^{N}\hat{\Sigma}^{-1}(\underline{x}_j-\hat{\underline{\mu}})(\underline{x}_j-\hat{\underline{\mu}})'$$

$$N = \hat{\Sigma}^{-1}\sum_{j=1}^{N}(\underline{x}_j-\hat{\underline{\mu}})(\underline{x}_j-\hat{\underline{\mu}})'$$

let $A = \sum_{j=1}^{N}(\underline{x}_j-\hat{\underline{\mu}})(\underline{x}_j-\hat{\underline{\mu}})'$

then

$$N = \hat{\Sigma}^{-1}A$$

$$\hat{\Sigma}N = \hat{\Sigma}\hat{\Sigma}^{-1}A$$

$$\hat{\Sigma}N = A$$

$$\boxed{\hat{\Sigma} = \frac{A}{N}} = \frac{\sum_{j=1}^{N}(x_j-\bar{x})(x_j-\bar{x})'}{N}$$

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we know that $\hat{\Sigma} = \frac{A}{N}$ is a biased estimator for Σ since

$$E(\hat{\Sigma}) = E\left(\frac{A}{N}\right) = \frac{1}{N}E\,\Sigma(x_j-\bar{x})(x_j-\bar{x})' = \frac{1}{N}E(\Sigma x_j x_j') - \bar{x}\bar{x}'$$

We Know that

$$\hat{\Sigma} = \frac{A}{N}$$

is abaised estimator for Σ since

$$E(\hat{\Sigma}) = E\left(\frac{A}{N}\right)$$

$$= \frac{1}{N} E \sum (\underline{x}_j - \bar{\underline{x}})(\underline{x}_j - \bar{\underline{x}})'$$

$$= \frac{E \sum x_j x_j' - N \bar{x}\bar{x}'}{N}$$

$$= \frac{\sum_{j=1}^{N} \hat{\Sigma} - N \frac{\hat{\Sigma}}{N}}{N}$$

$$= \frac{N\hat{\Sigma} - \hat{\Sigma}}{N}$$

$$\boxed{E(\hat{\Sigma}) = \frac{(N-1)}{N} \hat{\Sigma}}$$

Then the unbialed estimator is $S = \frac{A}{N-1}$ and S will be reffered to in the square as the Sample Covariance matrix.

ex: let $\underset{\sim}{X} \sim N_3(\underset{\sim}{\mu}, \Sigma)$

where $Q = X_1^2 + 2X_2^2 + \frac{1}{3}X_3^2 + 2X_1X_2 + 2X_2 - \frac{4}{3}X_3 + \frac{7}{3}$

Find $\underset{\sim}{\mu}$, Σ, k.

Hint $\left[\frac{\partial Q}{\partial \underset{\sim}{x}} = \mu, \ \Sigma = M^{-1}\right]$

$\boxed{P=3}$ نشتق ثلاثة مرات لانه توجد لدينا X_1، X_2، X_3.

$Q = X_1^2 + 2X_2^2 + \frac{1}{3}X_3^2 + 2X_1X_2 + 2X_2 - \frac{4}{3}X_3 + \frac{7}{3}$

$\frac{\partial Q}{\partial X_1} = 2X_1 + 2X_2 = 0$

$\therefore \boxed{X_1 = -X_2}$

$\frac{\partial Q}{\partial X_2} = 4X_2 + 2X_1 + 2 = 0$

$4X_2 + 2X_1 + 2 = 0$ but $X_1 = -X_2$ then

$4X_2 + 2(-X_2) + 2 = 0$

$4X_2 - 2X_2 + 2 = 0$

$2X_2 + 2 = 0$

$2X_2 = -2$

$\boxed{X_2 = -1}$

$\Rightarrow X_1 = -X_2 = -(-1) = 1$

$\boxed{X_1 = 1}$

where $\frac{\partial Q}{\partial \underset{\sim}{x}} = \begin{pmatrix} \frac{\partial Q}{\partial X_1} \\ \frac{\partial Q}{\partial X_2} \\ \frac{\partial Q}{\partial X_3} \end{pmatrix}$

$\frac{\partial Q}{\partial X_3} = \frac{2}{3}X_3 - \frac{4}{3}$

$0 = \frac{2}{3}X_3 - \frac{4}{3}$

$\frac{2}{\cancel{3}}X_3 = \frac{4}{\cancel{3}}$

$2X_3 = 4$

$\boxed{X_3 = 2}$

then we have $\underline{x} = \underline{\mu}$

Note

$x'x = x^2 \Leftarrow 2x$ المشتقة

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix}$$

$$\begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \underline{\mu}$$

نأخذ من ال ج.
نأخذ الحدود التي تمثل
مربعات المتغيرات
مثل x^2 وحاصل ضرب
المتغيرات مثل.
$x_1 x_2$

$$\underline{x}' M \underline{x} = x_1^2 + 2x_2^2 + \frac{1}{3} x_3^2 + 2x_1x_2$$

$$(x_1 \quad x_2 \quad x_3) \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1/3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\underline{x}' \quad M \quad \underline{x}$$

$$M = \Sigma^{-1}$$

then $\boxed{x'\Sigma^{-1}x = x'Mx}$

عندما نريد مصفوفة ال Σ^{-1}
هات
$\Sigma^{-1} = M$
وان $\leftarrow$

$$\Sigma = M^{-1} = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

بالنسبة ال M^{-1}

$$M^{-1} = \frac{1}{|M|} \operatorname{adj} M$$

$$\Rightarrow \quad x \sim N_3\left[\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}\right]$$

$$K = \frac{1}{(2\pi)^{3/2} |\Sigma|^{1/2}}$$

$=$

ex: let $X \sim N_3(\underline{\mu}, \Sigma)$

$f(x) = k \exp\{-\frac{1}{2}(x_1^2 + 5x_2^2 + 2x_3^2 + 2x_1x_2 + 3x_1x_3 - x_2x_3 - \frac{4}{3}x_3 + 4)$

find μ, Σ, k.

① $\Sigma^{-1} = \begin{pmatrix} 1 & 1 & 3/2 \\ 1 & 5 & -1/2 \\ 3/2 & -1/2 & 2 \end{pmatrix}$.

$\therefore \Sigma = \begin{pmatrix} 1 & 1 & 3/2 \\ & 5 & -1/2 \\ & & 2 \end{pmatrix}^{-1}$

$$\frac{\partial Q}{\partial X_1} = 2X_1 + 2X_2 + 3X_3 = 0$$

$$\frac{\partial Q}{\partial X_2} = 10X_2 + 2X_1 - X_3 = 0$$

$$\frac{\partial Q}{\partial X_3} = 4X_3 + 3X_1 - X_2 - \frac{4}{3} = 0$$

بحل هذه المعادلة نحصل على μ

$$\underline{\mu} = \underline{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \quad \end{pmatrix}$$

$$K = \frac{1}{(2\pi)^{3/2} |\Sigma|^{1/2}}$$

$=$

ex: let $x \sim N_3(\underline{\mu}, \Sigma)$

$f(x) = k \exp\{-\frac{1}{2}(x_1^2 + 5X_2^2 + 2X_3^2 + 2X_1X_2 + 3X_1X_3 - X_2X_3$

find μ, Σ, k.

① $\Sigma^{-1} = \begin{pmatrix} 1 & 1 & 3/2 \\ 1 & 5 & -1/2 \\ 3/2 & -1/2 & 2 \end{pmatrix}$.

$\therefore \Sigma = \begin{pmatrix} 1 & 1 & 3/2 \\ & 5 & -1/2 \\ & & 2 \end{pmatrix}^{-1}$

$$\frac{\partial Q}{\partial X_1} = 2X_1 + 2X_2 + 3X_3 = 0$$

$$\frac{\partial Q}{\partial X_2} = 10X_2 + 2X_1 - X_3 = 0$$

$$\frac{\partial Q}{\partial X_3} = 4X_3 + 3X_1 - X_2 - \frac{4}{3} = 0$$

بحل هذه المعادلة نحصل على μ

$$\underline{\mu} = \underline{X} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \quad \end{pmatrix}$$

$$K = \frac{1}{(2\pi)^{3/2} |\Sigma|^{1/2}}$$

$=$