"بسم الله الرحمن الرحيم"

<u>Cas(2) (2 - sample)</u>

let the indep. r. samples have been drawn from $MVN(\underline{\mu}, \Sigma)$, where Σ unknown and of full Rank p, the $100(1-\alpha)$ percent simultaneous Confidence intervals for all linear Compounds $\boxed{a'\delta}$ of the mean difference are defined by:

(To test $H_0: \underline{\mu}_1 = \underline{\mu}_2$).

$$Pr\left(a'(\bar{x}_1 - \bar{x}_2) - \sqrt{a'Sa\left(\frac{n_1+n_2}{n_1 n_2}\right)}\, T_{\alpha, p, n_1+n_2-p-1} \le a'\delta \le a'(\bar{x}_1 - \bar{x}_2) + \sqrt{a'Sa\left(\frac{n_1+n_2}{n_1 n_2}\right)}\, T_{\alpha, p, n_1+n_2} \right) =$$

since.

$$\delta = \underline{\mu}_1 - \underline{\mu}_2$$

and

$$T_{\alpha, p, n_1+n_2-p-1} = \sqrt{\frac{n_1+n_2-2}{n_1+n_2-p-1}\, F_{\alpha, p, n_1+n_2-p-1}}$$

<u>Note</u>

if Zero is included the intervals then we conclude that the means are not different at α.

(i.e. accept H_0).

ex) 2-samples are drawn with $n_1 =$ [illegible], $n_2 =$ [illegible]

and $\bar{x}_1 = \begin{bmatrix} 12.57 \\ 9.57 \\ 11.49 \\ 7.97 \end{bmatrix}$, $\bar{x}_2 = \begin{bmatrix} \text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \\ \text{[illegible]} \end{bmatrix}$ and $S_p =$ [illegible]

Test the hypothesis at $\alpha = 0.01$

① $H_0: \mu_{11} = \mu_{12}$

② $H_0: \mu_{11} + \mu_{21} = \mu_{12} + \mu_{22}$

note: $\mu_1 = \begin{pmatrix} \mu_{11} \\ \mu_{21} \\ \mu_{31} \\ \mu_{41} \end{pmatrix}$, $\mu_2 = \begin{pmatrix} \mu_{12} \\ \mu_{22} \\ \mu_{32} \\ \mu_{42} \end{pmatrix}$

$\underline{\text{Sol}}$ (1) $\quad \underline{\bar{x}}_1 - \underline{\bar{x}}_2 = \begin{pmatrix} 3.82 \\ 4.24 \\ 2.99 \\ 3.22 \end{pmatrix}$

$$a' S a = (1\ 0\ 0\ 0)\, S \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \boxed{11.25}$$

$$a'(\bar{x}_1 - \bar{x}_2) = 3.82$$

$$T_{0.01,\,4,\,44} = \sqrt{\frac{47(4)}{44} F_{0.01,4,44}}$$

$$= \sqrt{(4.272)(3.83)}$$

$$= \boxed{4.045}$$

$$Pr\left[3.82 - \sqrt{\frac{11.25(49)}{12(37)}}\,(4.045) \le \mu_{11} - \mu_{12} \le 3.82 + \sqrt{\frac{11.25(49)}{12(37)}}\,(4.045)\right] = 0.99$$

$$Pr\left(-0.687 \le \mu_{11} - \mu_{12} \le 8.327\right) = 0.99$$

$\therefore$ we accept H_0

$H_0: \mu_{11} = \mu_{12}$

sol (2)

$a' = (1\ 1\ 0\ 0)$

$$a'Sa = (1\,1\ 0\,0)\ S \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \boxed{43.58}$$

$$a'(\bar{x}_1 - \bar{x}_2) = (1\ 1\ 0\ 0)\begin{pmatrix} 3.82 \\ 4.24 \\ 12.99 \\ 3.22 \end{pmatrix} = \boxed{8.06}$$

$$Pr\left(8.06 - \sqrt{\frac{43.58(49)}{12(37)}}\,(4.045) \le \mu_{11} - \mu_{21} - \mu_{12} - \mu_{22} \le \right.$$

$$\left. 8.06 + \sqrt{\frac{43.58(49)}{12(37)}}\ 4.045\right) = 0.99$$

$$Pr\left(-0.8109 \le \mu_{11} - \mu_{41} - \mu_{12} - \mu_{22} \le 16.93\right) = 0.99$$

we accept H_0.

H.w

from the above example Test the hypothesis

(1) $H_0: \mu_{21} = \mu_{22}$

(2) $H_0: \mu_{31} + \mu_{41} = \mu_{32} + \mu_{42}$

(3) $H_0: \mu_{41} = \mu_{42}$

(4) $H_0: \mu_{31} = \mu_{32}$

Note: The simultaneous confidence interval (S.C.I) for the differences of the means at level $1-\alpha = 95\%$. calculating by the interval

$$(\bar{x}_{1j} - \bar{x}_{2j}) \mp \sqrt{\frac{(n_1+n_2)(n_1+n_2-2)}{(n_1 n_2)(n_1+n_2-p-1)} F_{(1-\alpha, p, n_1+n_2-p-1)} S_{jj}}$$

where $j = 1, 2, \ldots, p$

For example, the S.C.I for $\delta_1 = \mu_{11} - \mu_{21}$ is

$$(\bar{x}_{11} - \bar{x}_{21}) \mp \sqrt{\frac{(n_1+n_2)(n_1+n_2-2)}{(n_1 n_2)(n_1+n_2-p-1)} F_{(1-\alpha, p, n_1+n_2-p-1)} S_{11}}$$

where S_{11} is the element of the sample covariance matrix S_p.

H.W ① For the last example find

ⓐ S.C.I for $\mu_{12} - \mu_{22}$

ⓑ S.C.I for $\mu_{13} - \mu_{23}$

H.W ② Two psychological test wer given (5) mean and (4) woman. The data are recorded in the following table

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Man	x_1	25	21	26	19	28
	x_2	72	75	73	69	71
Women	x_1	18	27	22	26	
	x_2	71	73	65	78	

(a) Test the hypothesis:

$H_0: \underline{\mu}_1 = \underline{\mu}_2$ .vs $H_1\ \underline{\mu} \neq \underline{\mu}_2$ at $\alpha = 0.0$

(b) S.C.I for $\mu_{11} - \mu_{21}$

(c) S.C.I for $\mu_{12} - \mu_{22}$

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Correlation

The matrix of Sample ~~since~~ Correlation Can be Computed as:

$$R = D\left(\frac{1}{\sigma_i}\right) \Sigma D\left(\frac{1}{\sigma_i}\right)$$

or

$$R = D\left(\frac{1}{S_i}\right) S D\left(\frac{1}{S_i}\right)$$

or

$$R = D\left(\frac{1}{\sqrt{a_{ii}}}\right) A D\left(\frac{1}{\sqrt{a_{ii}}}\right)$$

where

$D(.)$ denotes the diagonal matrix Containing the reciprocal, of elements A.

Notes:

(1) R is p.s.d. or p.d. with diagonal elements.

(2) $-1 \leq r_{ij} \leq +1$

(3) r_{ij} are biased estimates.

(4) The matrix of partial correlation can be computed as:

$$R_{11.2} = D_{\Sigma_{11.2}}^{-\frac{1}{2}} \Sigma_{11.2} D_{\Sigma_{11.2}}^{-\frac{1}{2}}$$

or $$R_{11.2} = D_{S_{11.2}}^{-\frac{1}{2}} \left(S_{11} - S_{12} S_{22}^{-1} S_{21}\right) D_{S_{11.2}}^{-\frac{1}{2}}$$

$$= D_{S_{11.2}}^{-\frac{1}{2}} S_{11.2} D_{S_{11.2}}^{-\frac{1}{2}}$$

or $$R_{11.2} = D_{R_{11.2}}^{-\frac{1}{2}} \left(R_{11} - R_{12} R_{22}^{-1} R_{21}\right) D_{R_{11.2}}^{-\frac{1}{2}}$$

$$= D_{R_{11.2}}^{-\frac{1}{2}} R_{11.2} D_{R_{11.2}}^{-\frac{1}{2}}$$

(5) The dist. of r. if a r.s. of size N drawn from the bivariate normal population with $\rho = 0$ is:

$$P(r) = \begin{cases} \frac{1}{\sqrt{2\pi}} \frac{\Gamma\left(\frac{N-1}{2}\right)}{\Gamma\left(\frac{N-2}{2}\right)} (1-r^2)^{\frac{N-4}{2}} & , -1 < r < 1 \\ 0 & \text{o.w} \end{cases}$$

Which was ~~derived~~ derived by Fisher (1915

(6) The matrix of the partial Correlation $r_{ij;q+1,\dots,p}$ has the following dist.

$$P(r_{ij;q+1,\dots,p}) = \begin{cases} \frac{1}{\sqrt{2\pi}} \frac{\Gamma\left(\frac{N-q-1}{2}\right)}{\Gamma\left(\frac{N-q-2}{2}\right)} (1-r^2)^{\frac{N-q-4}{2}} & , -1 \leq r_{ij;q+1,\dots} \\ 0 & \text{o.w.} \end{cases}$$

where $r_{ij;q+1,\dots,p} = 0$

proof (b)

$$g(r) = \begin{cases} \frac{1}{\sqrt{2\pi}} \frac{\Gamma\frac{N-1}{2}}{\Gamma\frac{N-2}{2}} (1-r^2)^{\frac{N-4}{2}} & , -1 < r < 1 \\ 0 & \text{o.w.} \end{cases}$$

proof

we have t dist. with 2 d.f. is.

$$f(t) = \begin{cases} \frac{\Gamma\frac{N-1}{2}}{\Gamma\frac{N-2}{2}\sqrt{(n-2)\pi}} \left(1+\frac{t^2}{n-2}\right)^{-\frac{n-1}{2}} & , -\infty < t < \infty \\ 0 & \text{o.w.} \end{cases}$$

let $\boxed{t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}}$, $-1 < r < 1$

$$\therefore g(r) = f\left(r\frac{\sqrt{n-2}}{\sqrt{1-r^2}}\right) \cdot \left| r\frac{\sqrt{n-2}}{\sqrt{1-r^2}} \cdot \frac{1}{r} \right|$$

نعوض بدالة t بالعلاقة $\boxed{}$ فينتج

$$g(r) = \frac{\Gamma\frac{n-1}{2}}{\Gamma\frac{n-2}{2}\sqrt{n-2}\,\Gamma\frac{1}{2}} \cdot \left(1+\frac{r^2\cancel{(n-2)}}{(1-r^2)\cancel{(n-2)}}\right)^{-\frac{n-1}{2}} \cdot \left| \frac{\sqrt{n-2}\sqrt{1-r^2} + \frac{\sqrt{n-2}}{\sqrt{1-r^2}} r^2}{(1-r^2)} \right|$$

$$= \frac{\Gamma\frac{n-1}{2}}{\Gamma\frac{1}{2}\,\Gamma\frac{n-2}{2}\sqrt{n-2}} \left(1+\frac{r^2}{1-r^2}\right)^{-\frac{n-1}{2}} \cdot \frac{\sqrt{n-2}\,(1-r^2) + \sqrt{n-2}\,r^2}{(1-r^2)^{3/2}}$$

$$= \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n-2}{2}\right)\sqrt{n-2}\,\Gamma\left(\frac{1}{2}\right)} \left(\frac{1-\cancel{r^2}+\cancel{r^2}}{1-r^2}\right)^{-\frac{n-1}{2}} \left(\frac{\sqrt{n-2}\,(1-r^2)+\sqrt{n-2}\,r^2}{(1-r^2)^{3/2}}\right)$$

$$= \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n-2}{2}\right)\cancel{\sqrt{n-2}}\,\Gamma\left(\frac{1}{2}\right)} \left(\frac{1}{1-r^2}\right)^{-\frac{n-1}{2}} \left(\frac{\cancel{\sqrt{n-2}}\,[1-r^2+r^2]}{(1-r^2)^{3/2}}\right)$$

$$= \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{n-2}{2}\right)} \left((1-r^2)^{-1}\right)^{-\frac{n-1}{2}} \left(1-r^2\right)^{-3/2}$$

$$= \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n-2}{2}\right)\Gamma\left(\frac{1}{2}\right)} (1-r^2)^{\frac{n-1}{2}} (1-r^2)^{-3/2}$$

$$= \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n-2}{2}\right)\Gamma\left(\frac{1}{2}\right)} (1-r^2)^{\frac{n-1}{2}-\frac{3}{2}}$$

$$\boxed{g(r) = \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n-2}{2}\right)\Gamma\left(\frac{1}{2}\right)} (1-r^2)^{\frac{n-4}{2}}}$$

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Recall
t-distribution with n.d.f.

p.340 Hogg [illegible]

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(7) if $\rho \neq 0$ the dist. of r has acomplicated form, So Fisher (1921) showed that the monotonic transformation for Large Samples

$$Z = \frac{1}{2} \ln \frac{1+r}{1-r} = \tanh^{-1} r$$

$$Z = \tanh^{-1}(r) \sim \text{Asymptotically normal}$$

with mean

$$E(z) \simeq \frac{1}{2} \ln \frac{1+\rho}{1-\rho}$$

and variance

$$Var(z) = \frac{1}{N-3}$$

Values of z transformation are given in table (5) of the Morrison Appendix P(370)

(8) (a) To test the hypothesis

$H_0: \rho = 0$

v.s. $H_1: \rho \neq 0$

with the prop. α by the rule.

Accept H_0 if $|r| \sqrt{\frac{N-2}{1-r^2}} \ngtr t_{(\frac{\alpha}{2}, N-2)}$

Accept H_1 if $|r| \sqrt{\frac{N-2}{1-r^2}} < t_{(\frac{\alpha}{2}, N-2)}$

Where $t = r\sqrt{\frac{N-2}{1-r^2}}$ is student fisher t-dist. with $(N-2)$ d.f.

(b) To test the hypothesis

$H_0: \rho = 0$

v.s. $H_1: \rho > 0$

$H_1: \rho > 0$

The decision Rule would be.

Accept H_0: if $r\sqrt{\frac{N-2}{1-r^2}} \leq t(\alpha, N-2)$

Accept H_1: if $r\sqrt{\frac{N-2}{1-r^2}} > t(\alpha, N-2)$

(c) To test the hypothesis

$H_0: \rho = 0$ v.s. $H_1: \rho < 0$

The decision rule would be.

accept H_0 if $r\sqrt{\frac{N-2}{1-r^2}} \geq -t_{\alpha, N-2}$

accept H_1 if $r\sqrt{\frac{N-2}{1-r^2}} < -t_{\alpha, N-2}$

(9) To test the hypothesis

$H_0: \rho = \rho_0$

v.s. $H_1: \rho \neq \rho_0$

$r = \rho_0$
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with prop.(α), the decision rule is:

accept H_0 if $|z - \zeta_0|\sqrt{N-3} \leq Z_{\frac{\alpha}{2}}$

accept H_1 if $|z - \zeta_0|\sqrt{N-3} > Z_{\frac{\alpha}{2}}$

where ζ_0 is the Z transformation

(10) The C.I. for ρ is.

$$\tanh^{r}\left(z - \frac{Z_{\alpha/2}}{\sqrt{N-3}}\right) \leq \rho \leq \tanh^{r}\left(z + \frac{Z_{\frac{\alpha}{2}}}{\sqrt{N-3}}\right)$$

(11) if two independent r.s. of sizes n_1 and n_2 have been drawn from bivariate normal populations To test,

$H_0: \rho_1 = \rho_2$

v.s. $H_1: \rho_1 \neq \rho_2$

Can be made by applying the z-transformation and computing:

$$d = \frac{z_1 - z_2}{\sqrt{\frac{1}{n_1 - 3} + \frac{1}{n_2 - 3}}}$$

Then the decision rule would be:

accept H_0: if $|d| \leq z_{\alpha/2}$

accept H_1: if $|d| > z_{\alpha/2}$

<u>Note</u> Rao (1965, pp (364 - 368)

has offered a test of equality of the population correlations of k-indep. Samples

(12) The power fun. of the single Sample test of $H_0: \rho = \rho_0$ v.s. $H_1: \rho \neq \rho_0$ is.

$$1 - \beta(\rho) = \Phi\left[-Z_{\frac{\alpha}{2}} + (\zeta_0 - \zeta)\sqrt{N-3}\right] + 1 - \Phi\left[Z_{\frac{\alpha}{2}} + (\zeta_0 - \zeta)\sqrt{N-3}\right]$$

where $\Phi(.)$ is the standard normal dist. fun.

Similarly the power fun. for the α test. of $H_0: \rho = \rho_0$, v.s. $H_1: \rho > \rho_0$ (against the one sided alternative) and H_1 $\rho < \rho_0$

means

$H_0: \rho = \rho_0$

v.s $H_1: \rho > \rho_0$

$H_1: \rho < \rho_0$

are

$$1 - \beta(\rho) = 1 - \Phi\left[Z_\alpha + (\zeta_0 - \zeta)\sqrt{N-3}\right]$$

$$1 - \beta(\rho) = \Phi\left[-Z_\alpha + (\zeta_0 - \zeta)\sqrt{N-3}\right]$$

respectively

exp. 3.2 p. 106 in Morrison.

(13) To test $H_0: R = I$

$H_1: R \neq I$

its equivalent that Σ is adigonal matrix when the variates are multinormal, the test is one of complete indep. of the responses we use the following exact Bartlet statistic (1954) test:

$$\chi^2 = -\left(N-1-\frac{2p+5}{6}\right)\ln|R|$$

So we accept H_0 if:

$$\chi^2 < \chi^2_{\alpha, \frac{p(p-1)}{2}}$$

and we use the following approximation lowley (1940) test (for small correlations):

$$\chi^2 \cong \left(N-1-\frac{2p+5}{6}\right)\sum\sum_{i<j} r_{ij}^2$$

exp. 3.5 p.119 Morrison.

Cas A

Σ معلومة

(i) univariat

one sample. to test $H_0: \mu = \mu_0$

$\sqrt{n}(\bar{x} - \mu_0) \sim N(0, \sigma^2)$

[illegible]

two sample.

to test $H_0: \mu_1 = \mu_2$

$\sqrt{\frac{n_1 n_2}{n_1+n_2}} (\bar{x}_1 - \bar{x}_2) \sim N(0, \sigma^2)$ [illegible]

~~[illegible]~~

(ii) multivariate.

two-sample

$\frac{n_1 n_2}{n_1+n_2} (\bar{x}_1 - \bar{x}_2)' \Sigma^{-1} (\bar{x}_1 - \bar{x}_2) \sim \chi^2_p$

to test

$H_0: \mu_1 = \mu_2$

$\frac{n_1 n_2}{n_1+n_2} (\bar{x}_1 - \bar{x}_2)' \Sigma^{-1} (\bar{x}_1 - \bar{x}_2) \geq \chi^2_p$

we reject H_0.

$\frac{n_1 n_2}{n_1+n_2} D^2 \geq \chi^2_p$ $D^2 = (\mu_1 - \mu_2)' \Sigma^{-1} (\mu_1 - \mu_2)$

one sample.

$\sqrt{n}(\bar{x} - \mu_0) \sim N(0, \Sigma)$

[illegible]

$n(\bar{x} - \mu_0)' \Sigma^{-1} (\bar{x} - \mu_0) \sim \chi^2_p$

to test $H_0: \mu = \mu_0$

$n(\bar{x} - \mu_0)' \Sigma^{-1} (\bar{x} - \mu_0) \geq \chi^2_{(p)}$

we reject H_0.

Case B

Σ unknown

Σ غير معروفة وكذلك μ.

(i) univarit

[illegible]

$\lambda = \frac{L(\hat{\omega})}{L(\hat{\Omega})}$

$t^2 = (n-1)(\lambda^{-2/n} - 1)$

$|t| \leq k_0$

the critical region

(ii) multivariate (one sample)

$H_0: \mu = \mu_0$

$\omega = [\mu = \mu_0, \hat{\Sigma} = \frac{\Sigma(x_\alpha - \mu_0)(x_\alpha - \mu_0)'}{n}]$

$\Omega = (\hat{\mu} = \bar{x}, \hat{\Sigma} = \frac{\Sigma(x_\alpha - \bar{x})(x_\alpha - \bar{x})'}{n}]$

$\lambda = \frac{L(\hat{\omega})}{L(\hat{\Omega})}$

$\lambda^{2/n} \leq \lambda_0^{2/n}$

$T^2 \geq (n-1)(\lambda_0^{-2/n} - 1)$

$T^2_{cal} \geq T^2_{table}$

$T^2_{table} = \frac{(n-1)p}{n-p} F_{p, n-p, \alpha}$

~~$T^2_{cal} = n(\bar{x} - \mu_0)' S^{-1} (\bar{x} - \mu_0)$~~

$T^2_{cal} = n(\bar{x} - \mu_0)' S^{-1} (\bar{x} - \mu_0)$

(iii) [illegible]

Σ غير معروفة (2-Sample)

$H_0: \mu^{(1)} = \mu^{(2)}$

$\boxed{T^2_{cal} \geq T_0}$ we reject H_0.

$T_0^2 = \frac{(n_1+n_2-2)p}{n_1+n_2-p-1} F_{p, n_1+n_2-p-1, \alpha}$

$T^2_{cal} = \frac{n_1 n_2}{n_1+n_2} (\bar{x}^{(1)} - \bar{x}^{(2)})' S^{-1} (\bar{x}^{(1)} - \bar{x}^{(2)})$

معروفات.

① $\Sigma_1 = \Sigma_2 \longrightarrow$ (iii) (B) with $2n-2$ df

② $n_1 < n_2$

$H_0: \mu^{(1)} = \mu^{(2)}$ $\quad n_1 \bar{y}' S^{-1} \bar{y} \sim T^2_{(n_1-1)}$

$$S = \frac{\sum \left(X_\alpha^{(1)} - \bar{X}_\alpha^{(1)} - \sqrt{\frac{n_1}{n_2}} \left(X_\alpha^{(2)} - \frac{1}{n_1} \sum X_\beta^{(2)}\right)\right)(\;)'}{n_1 - 1}$$

③ $n_2 < n_1$ ✓

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Case A

Σ : known

Σ معروفة

Case C

When have sample $\left(X_\alpha^{(1)}\right)$ $\quad \alpha = 1, 2, \dots, n_1$

(i)

$n_1 = n_2 = n$

$(\Sigma_1$ & Σ_2 known$)$ معروفة

$n \bar{y}' S^{-1} \bar{y} \geq \chi^2_p$

$n(\bar{x}^{(1)} - \bar{x}^{(2)})' (\Sigma_1 + \Sigma_2)^{-1} (\bar{x}^{(1)} - \bar{x}^{(2)}) \geq \chi^2_p$

To test $H_0: \mu^{(1)} = \mu^{(2)}$ or $H_0: \mu^{(1)} - \mu^{(2)} = 0$ or $H_0: \nu = 0$

(ii)

$n_1 = n_2 = n$ $(\Sigma_1$ and Σ_2 [illegible]$)$

$T^2 = n \bar{y}' S^{-1} \bar{y} \sim T^2_{(n-1)}$

$n(\bar{x}^{(1)} - \bar{x}^{(2)})' S^{-1} (\bar{x}^{(1)} - \bar{x}^{(2)}) \sim T^2_{(n-1)}$

$$S = \frac{\sum (y_\alpha - \bar{y}_\alpha)(y_\alpha - \bar{y}_\alpha)'}{(n-1)}$$

$$\therefore S = \frac{\sum \left((x_\alpha^{(1)} - x_\alpha^{(2)}) - (\bar{x}^{(1)} - \bar{x}^{(2)})\right)(\;)'}{(n-1)}$$

Properties T^2

Case A

$x_1, x_2, \ldots, x_n \sim N_p(\mu, \Sigma)$

Σ [illegible]

$H_0: \mu = \mu_0$

① $T^2 = n(\bar{x} - \mu_0)' S^{-1} (\bar{x} - \mu_0) \sim T^2$

$S = \dfrac{\Sigma (x_\alpha - \bar{x}_\alpha)(x_\alpha - \bar{x}_\alpha)'}{n-1}$ unbiased for Σ

② $T^2 = \dfrac{|S + n(\bar{x} - \mu_0)(\bar{x} - \mu_0)'|}{|S|} - 1$

③ the dist

$u = \dfrac{(n-p)}{(n-1)(p-1)} T^2 = F'_{p-1, n-p, \alpha}$

non central

④ $T^2_{cal} > T_0$

$H_0: \mu = \mu_0$

GLRT for

is

Cas B

$X_\alpha^{(i)} \quad \alpha = 1, 2, \ldots n_i \sim N(\mu^{(i)}, \Sigma) \quad i = 1, 2$

$\mu^{(1)} = \mu^{(2)}$

The test based on

① $\dfrac{n_1 n_2}{n_1 + n_2} (\bar{x}^{(1)} - \bar{x}^{(2)})' S^{-1} (\bar{x}^{(1)} - \bar{x}^{(2)}) \sim T^2_{n_1 + n_2 - 2}$

$S = \dfrac{\Sigma (x_\alpha^{(1)} - \bar{x}_\alpha^{(1)})(x_\alpha^{(1)} - \bar{x}_\alpha^{(1)})' + \Sigma (x_\alpha^{(2)} - \bar{x}_\alpha^{(2)})(x_\alpha^{(2)} - \bar{x}_\alpha^{(2)})'}{n_1 + n_2 - 2}$

② The dist.

$u = \dfrac{n_1 + n_2 - p - 1}{(n_1 + n_2 - 2)(p - 1)} T^2 = F'_{p-1, n_1 + n_2 + p - 1, \alpha}$

③ GLRT. $H_0: \mu^{(1)} = \mu^{(2)}$

$T^2 \geq T_0^2$

Cas C

$X_\alpha^{(i)}, \alpha = 1, 2, \ldots, n \sim N(\mu^{(i)}, \Sigma_i)$

$H_0: \mu^{(1)} = \mu^{(2)}$

① $\bar{x}^{(i)} \sim N_p(\mu^{(i)}, \frac{1}{n_i} \Sigma_i)$

② $\bar{Y} \sim N_p(0, \frac{\Sigma_Y}{n})$ $\quad \Sigma_Y = \Sigma_1 - \Sigma_2$

$Y_\alpha = X_\alpha^{(1)} - X_\alpha^{(2)}$

③ to test $H_0: \mu^{(1)} - \mu^{(2)} = 0$

$n \bar{Y}' S^{-1} \bar{Y} \sim T^2(n-1)$

$S = \dfrac{\Sigma (Y_\alpha - \bar{Y}_\alpha)(Y_\alpha - \bar{Y}_\alpha)'}{n-1}$

Simulation inferences for mean.

Case 1

one Sample

$\boxed{H_0: a'\mu = a'\mu_0}$

$$P_r\left(a'\bar{X} - \sqrt{\frac{a'Sa}{n}}\, T_{(n-p),p,\alpha} \leq a'\mu \leq \quad + \quad\right) = 1-\alpha$$

$$T_{\alpha,p,n-p} = \sqrt{\frac{(n-1)p}{n-p} F_{p,n-p,\alpha}}$$

Case 2

2 - Sample

$\mu_1 = \mu_2$

$$P_r\left(a'(\bar{X}_1 - \bar{X}_2) - \sqrt{a'Sa\frac{n_1+n_2}{n_1 n_2}}\, T_{p,n_1+n_2-p-1,\alpha} \leq a'\delta \leq \quad + \quad\right) = 1-\alpha$$

$$\delta = \mu_1 - \mu_2$$

$$\mu_1 = \begin{pmatrix} \mu_{11} \\ \mu_{21} \\ \mu_{31} \\ \mu_{41} \end{pmatrix} \quad \mu_2 = \begin{pmatrix} \mu_{12} \\ \mu_{22} \\ \mu_{32} \\ \mu_{42} \end{pmatrix}.$$

$$T_{\alpha,p,n_1+n_2-p-1} = \sqrt{\frac{(n_1+n_2-1)p}{n_1+n_2-p-1} F_{p,n_1+n_2-p-1}}$$