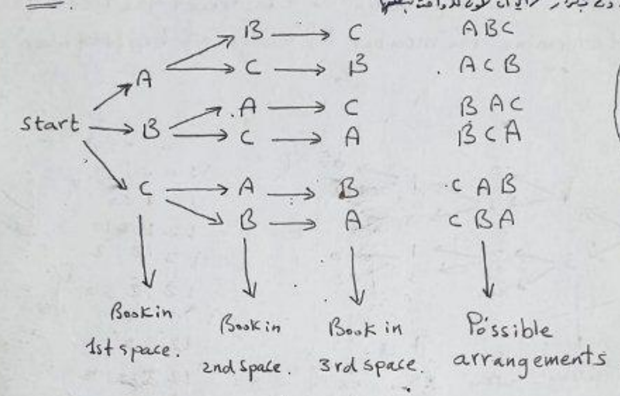
Tree Diagrams

ex. 24. In How many ways can three books denoted by A, B, C be arranged in order on aself?

Sol.

بكم طريقة يمكن ترتيب ثلاثة كتب A, B, C .
حيث تكون بدون تكرار أي أن تكون كل واحدة بنفسها

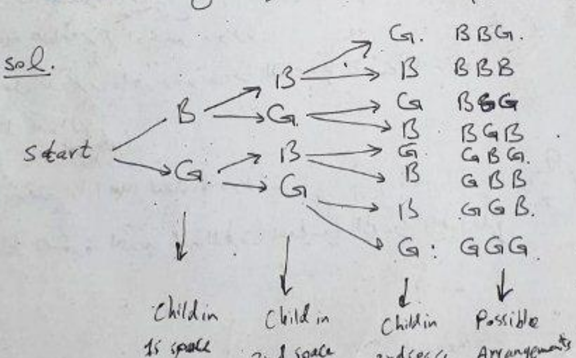


بكم طريقة يمكن ترتيب ثلاثة كتب A, B, C.
بحيث تكون بدون تكرار أي كل واحدة منفردة

عدد الطرق = 6

ex. 25 Suppose that the cuple is planning to have three children. In how many ways can this happen?

Sol.



زوجين قررا انجاب ثلاثة
اطفال. بكم طريقة يمكن
ترتيب الاطفال
علماً ان يكون لديهم ولد وبنت وغيرها.

هنا يجوز التكرار لانه لم يحدد

عدد الطرق = 8

Remark: The number of arrangments of n different objects in a row is given by $\boxed{n!}$.

قاعدة 2/ عدد ترتيب n من الأشياء المختلفة في صف هو $n!$. (مفكوك n)

Remark: The number of arrangment of n different objects in a circle is given by:

$$(n-1)!\ ,\ n \geq 1$$

نتيجة: عدد ترتيب n من الأشياء المختلفة حول دائرة هو $(n-1)!$.

مثال 7/ بكم طريقة يمكن غرس 4 أشجار مختلفة حول حديقة دائرية؟

$$(n-1)! = (4-1)! = 3! = \boxed{6} \text{ طرق}$$

ex. 29: What is the number of ways in which 6 pearsons can be seated in a circle?

$$(n-1)! = (6-1)! = 5! = \boxed{120}$$

can be seated in a row.

$n! = 6! =$

ex 30) What is the number of ways in which 6 books can be arranged (i) side by side? (ii) in a row (iii) around circle.

(i) **sol** $n! = 6! = \boxed{720}$

يعني بشكل صف

or

6	5	4	3	2	1

$6 * 5 * 4 * 3 * 2 * 1 = \boxed{720}$

(ii) in a row

(ii) نفسه

(iii) in a circle.

$(n-1)! = (6-1)! = 5! = 5*4*3*2*1 = \boxed{120}$

(i) with repetition (replacement)

6	6	6	6	6	6

$6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^6 =$

ex. 31) What is the number of ways in which 5 persons can be seated in a row if a certain (2) of them must sit side by side?

sol. $n! = 5! = 5 * 4 * 3 * 2 * 1 = \boxed{120}$

$120 * 2 = \boxed{240}$ ways

(ii) in a circle $(n-1)! = (5-1)! = 4! = 4 \times 3 \times 2 \times 1 = \boxed{24 \text{ ways}}$

كم طريقة ان يجلس 6 اشخاص بشكل صف على ان يكون كل اثنين بجانب بعض.

الحل / نحسب كم طريقة للصف وهو $n!$

ولكل اثنين يكون $\boxed{2 * n!}$

circle

row

(i) with replacement)

5	5	5	5	5	

$5 \times 5 \times 5 \times 5 \times 5 = 5^5 =$

Factorial Notation

The product of the positive integers from 1 to n inclusive occurs very often in mathematics and hence is denoted by the special symbol n! (read n factorial)

① $$\boxed{n! = 1.2.3.4.\ldots(n-2)(n-1)n.}$$

$n(n-1)(n-2)(n-3)\ldots 3.2.1$

It is also convenient to define $0! = 1$.

توضيح من انا لمقة

$$\begin{cases} 0! = 0(0-(-1))(-1-(-2))(-2-(-3))\cdots(-(n-1)-(-n)) \\ \quad = (1)(1)(1)\cdots(1) \\ \quad = 1 \end{cases}$$

② **example**

$2! = 2 \cdot 1 = 2$

$3! = 3 \cdot 2 \cdot 1 = 6 \Rightarrow 3 \cdot 2! = 6$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24 \Rightarrow 4 \cdot 3! = 24$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5 \cdot 4! = \boxed{120}$

③ **example**

$$\frac{8!}{6!} = \frac{8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!}} = \boxed{56}$$

$$\frac{12!}{9!} = \frac{12 \cdot 11 \cdot 10 \cdot \cancel{9!}}{\cancel{9!}} = \boxed{1320}$$

بسم الله الرحمن الرحيم

Permutation التباديل .

Def. 15 $\boxed{\text{permutation}}$

A permutation of anumber of objects is any arrangment of these objects in adefinite order.

التبديل لعدد من الاشياء (مجموعة) هو اي ترتيب لها في وضع محدد .

Def. 16 : $\boxed{\text{permutation}}$

An arrangment of r objects, taken from aset of n objects, is called apermutation of the n objects, taken r at atime The total number of such permutation is denoted by:

$$P_r^n \ni 0 \le r \le n$$

الترتيب لـ r من المفردات المختلفة مأخوذة من مجموعة اكبر تمثل n من المفردات . (التبديل) تدعى تبديل حيث $(r \le n)$ وان العدد الكلي للتباديل يرمز له بالرمز P_r^n حيث ان n عدد مفردات المجموعة و r عدد المفردات المأخوذة للترتيب , P تعني تباديل permutation والجمع لها.

$$P_r^n = \frac{n!}{(n-r)!}$$

التباديل للأحرف a, b, c, d وبشكل مختلف.

ex. Consider the set of letters a, b, c, d then we have

(i) b d c a , d c b a , a c d b , ... it is a permutation of 4 letters (taken all at time)

$P_r^n = P_4^4 = \frac{4!}{0!} = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{24}$ نلاحظ هنا وجود ٢٤ تبديل حيث

(ii) b a d , a d b , c b d , b c a " " " " (" 3 at time)

$P_r^n = P_3^4 = \frac{4!}{(4-3)!} = 4! = \boxed{24}$ اخذها ٢٤ تبديل.

(iii) a d , c b , d a , b d " " " " (" 2 " time)

$P_r^n = P_2^4 = \frac{4!}{(4-2)!} = \frac{4!}{2!} = 4 \cdot 3 = \boxed{12}$ اخذها عدد التباديل

Theorem 14 . For Permutation

1- ((Permutation of n things, all together)) (التباديل لـ n من المفردات مأخوذة كلها بنفس الوقت). $n = r$

The number of permutation of n different objects taken all at time

$\boxed{n!}$

(التباديل لـ n من المفردات كلها مع بعض) بنفس الوقت.

(عدد التراتيب والتباديل) لـ n من المفردات المختلفة مأخوذة كلها مرة واحدة هو $\boxed{n!}$.

where.
$$P_n^n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = \boxed{n!}$$

$n \neq r$

2- (Permutation of n things, taken r at time) (التباديل لـ n من المفردات مأخوذة r في كل مرة)

The number of permutation of n different objects taken r at time, is. (P_r^n) and is defined as ((without repetitions)).

$$\boxed{P_r^n = \frac{n!}{(n-r)!}} , \quad 0 \leq r \leq n \quad \text{where } P_r^n = P(n,r) = n(n-1)(n-2)\dots(n-r+1)$$
$$= \frac{n!}{(n-r)!}$$

عدد التباديل لـ n من المفردات المختلفة مأخوذة r في كل مرة. هي P_r^n

وصيغتها مع عدم وجود التكرار هي

Theorem.

$$P^n_r = P(n,r) = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

The second part of the formula follows from the fact that.

$$n(n-1)(n-2)\cdots(n-r+1) = \frac{n(n-1)(n-2)\cdots(n-r+1)\cancel{(n-r)}!}{\cancel{(n-r)}!} = \frac{n!}{(n-r)!}$$

In special case that $r=n$ we have

$$P^n_r = P^n_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = \boxed{n!}.$$

where $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1) = n!$

ex. 20 How many 2 digit number can be formed from the seven digit numbers ?. If repitions is not allowed.
(1, 2, 3, 4, 5, 6, 7) with replacment

كم طريقة يمكن صياغة عدد يتكون من رقمين
باستخدام الأرقام (1,2,3,4,5,6,7) على أن لا يتكرر

Sol: 1st method

7	6

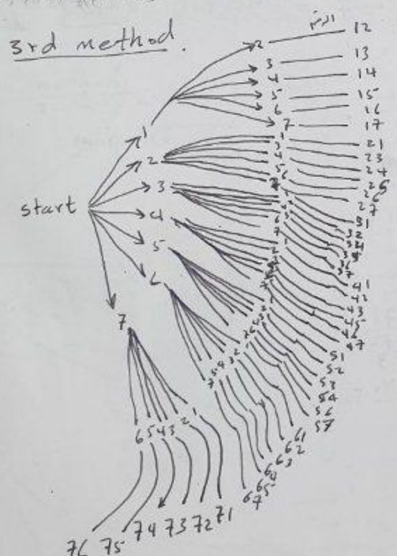
$7 * 6 = 42$ ways.

2nd method

$n = 7$
$r = 2$.

$$\Rightarrow P^7_2 = \frac{n!}{(n-r)!} = \frac{7!}{(7-2)!} = \frac{7!}{5!} = \frac{7*6*\cancel{5!}}{\cancel{5!}} = \boxed{42}$$

3rd method.



الطرق المذكورة هنا.
1st - طريقة الترتيب Arrangement
2nd - طريقة التباديل Permutation
3rd - طريقة شجرة Tree Diagram

ex. 28. How many arrangements can be made of the letters of the word (Mississippi), taken all together?

sol

since ther are. $m = 1$, $i = 4$, $s = 4$, $p = 2$

and the total of letters is $\boxed{11}$, then.

$$\frac{n!}{n_1!\, n_2!\, n_3! \ldots n_k!} = \frac{11!}{1!\, 4!\, 4!\, 2!} = \boxed{34650}$$

How many arrangment. Can be mad of the letters of the words $\boxed{\text{statistics}}$ taken all together

سؤال/. ما عدد التراتيب المختلفة التي يمكن تكوينها من حروف كلمة statistics

اذا اخذت جميعها.

الكلمة تحتوي على.

$S = 3$, $t = 3$, $i = 2$, $a = 1$, $c = 1$

وبذلك فان $K = 5$ و

$n_1 = 3$, $n_2 = 3$, $n_3 = 2$, $n_4 = 1$, $n_5 = 1$

$$n_1 + n_2 + n_3 + n_4 + n_5 = 3 + 3 + 2 + 1 + 1 = \boxed{10}$$

وعلى هذا الاساس فان عدد التراتيب

$$\frac{n!}{n_1!\, n_2!\, n_3!\, n_4!\, n_5!} = \frac{10!}{3!\, 3!\, 2!\, 1!\, 1!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3! \times 3 \times 2 \times 1 \times 2 \times 1 \times 1}$$

$$= \boxed{50400}$$

عدد التراتيب او الطرق.

ex. use world $\boxed{\text{Baghdad}}$.

ex. 27: How many 4 digits numbers can be formed from the digit numbers 1, 2, 3, 4 ? repition is not allowed.

1st method.

4	3	2	1

$4 * 3 * 2 * 1 = 24$

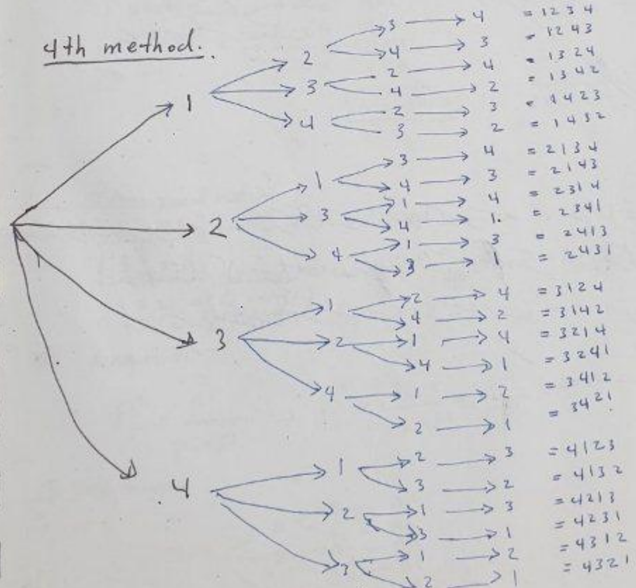
2nd method.

$n! = 4! = 4 \times 3 \times 2 \times 1 = \boxed{24}$

3rd method.

$$P^n_r = P^4_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4!}{1} = 4 \times 3 \times 2 \times 1 = \boxed{24}$$

4th method.



H.W with replecment.? (H.W)

EX (3)

In How many ways can three Books denoted by, A, B and C

(i) be arranged in order on aself ? ترتيبهم شخصيا (عدد ترتيبات)

(ii) bearranged only 2 letters ? ترتيب فقط حرفين منهم

(iii) with replecment . for (i)

(iv) with respect 2 only letters with replecment .

(i) ① arranged . method

1	2	3
3	2	1

$3 \times 2 \times 1 = \boxed{6.}$ ways.

② $P^n_n = P^3_3 = 3! = 3 * 2 * 1 = \boxed{6}$ ways . يرتب بكلهم

or $P^n_n = n! = 3! = \boxed{6}$

③ Tree Diagram.

start
- A
  - B — C   ABC
  - C — B   ACB
- B
  - A — C   BAC
  - C — A   BCA
- C
  - A — B   CAB
  - B — A   CBA

$\boxed{\text{عدد الطرق} = 6}$

(ii) ① Arrangment method.

①

1	2
3	2

$3 * 2 = \boxed{6 \text{ ways}}$

② permutation

$$P^n_r = \frac{n!}{(n-r)!} \Rightarrow P^3_2 = \frac{3!}{(3-2)!} = \frac{3!}{1!} = 3*2*1 = 6 \boxed{\text{ways}}$$

③ Tree Diagram

start
- A
  - B — AB
  - C — AC
- B
  - A — BA
  - C — BC
- C
  - A — CA
  - B → CB

$\boxed{\text{عدد الطرق} = 6}$

(iii). with replecment.

① $P \Rightarrow n^r = 3^3 = \boxed{27}$ ways

التباديل

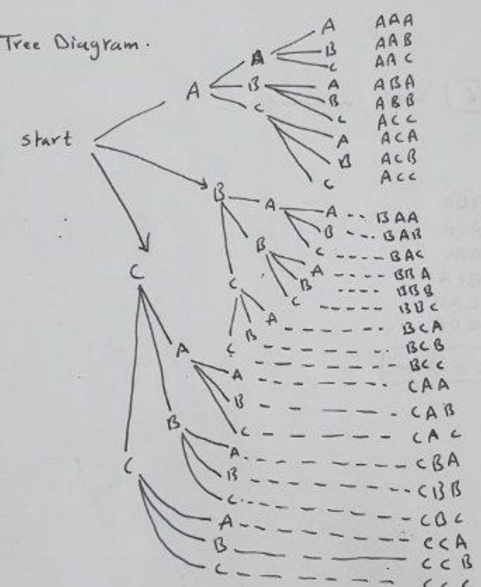
~~or ③ $P^n_r + P^n_r = P^n_r + n = P^3_2 + 3 = 6 + 3 = 9$~~

② Arrangment method.

1	2	3
3	3	3

$3 \times 3 \times 3 = 3^3 = \boxed{27}$ ways.

③ Tree Diagram.



$\boxed{27}$ ways

iv) 2 only letters with replecmant.

① Arrangment method.

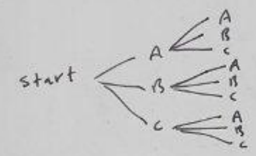
1	2
3	3

$3 * 3 = \boxed{9}$ ways.

② permutation

$n^r = 3^2 = \boxed{9}$ ways

③ Tree Diagrams.



AA
AB
AC
BA
BB
BC
CA
CB
CC

$\boxed{9 = \text{عدد الطرق}}$

④

كم عدد يتكون من رقمين ممكن تكوينه من الارقام

How many 2-digit numbers can be formed from the digits (2, 4, 5, 7)

if (i) Repetation are not allowed. (ii) with replacement.

(iii) 4-digit only. without replacement.

(IV) with replacement.

① التكرار غير مسموح

② وجود التكرار

③ تكوين عدد يتكون من 4 ارقام بدون تكرار من هذه الارقام

④ مع وجود التكرار

(i)

① | 4 | 3 |

$4 * 3 = \boxed{12}$ ways

② $P^n_r = P^4_2 = \frac{4!}{(4-2)!} = \frac{4!}{2!} = 4*3 = \boxed{12}$ ways.

③

24
25
27
42
45
47
52
54
57
72
74
75

$= \boxed{12 \text{ ways}}$

(ii) with replacement.

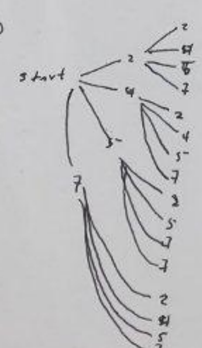
① | 4 | 4 |

$4 * 4 = 16$ ways

②

$n^r = 4^2 = 4*4 = \boxed{16}$ ways.

③



$= \boxed{16 \text{ ways}}$

7) (iii) 4-digit only without replacement

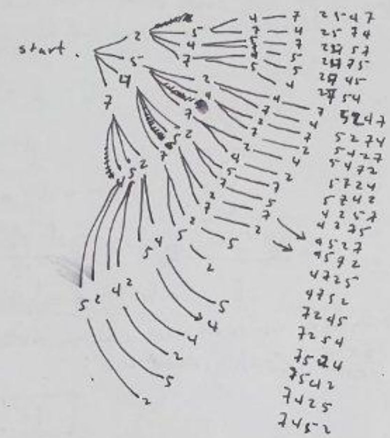
① | 4 | 3 | 2 | 1 |

$4 * 3 * 2 * 1 = \boxed{24}$ ways.

② $P^n_n = \frac{n!}{0!} = n! = 4! = 4*3*2*1 = \boxed{24 \text{ ways}}$

or $P^n_n = n! = 4! = 24$.

③ tree diagram.



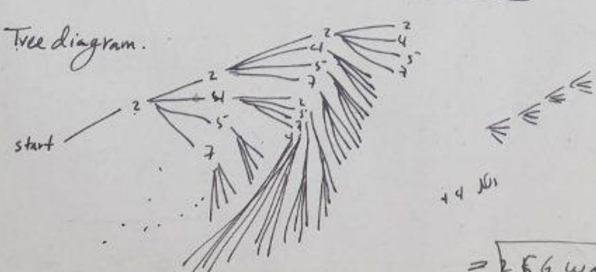
(iv) With replacement. 4-digit only

① | 4 | 4 | 4 | 4 |

$4*4*4*4 = 16*16 = \boxed{256 \text{ ways}}$.

② $n^r = 4^4 = 4*4*4*4 = 16.16 = \boxed{256 \text{ ways}}$.

③ Tree diagram.



$= \boxed{256 \text{ ways}}$

سؤال خارجي

Ex The number 1, 2, 3 and 4 are written seperately on four slips of paper. The slips are then put into a hat and mixed. A pearson draws two slips from the hat, one after the other without replacement. Find the number of ways?

Ex In how many ways can 3 white, 4 red and 4 black balls be arranged in a row if similarly Colored (ملونة) balls are unordered from each other.

$$\frac{11!}{3! \cdot 4! \cdot 4!}$$

7 (1)

How many 3-digit numbers can be formed from the digits (2, 3, 9, 5, 6, 7)
(i) without replacement.
(ii) the numbers that less than 400 ?
(iii) if the number is even.
(iv) if the number is ~~constant the hundred~~ multipl by 5 ?
(v) if the number is odd.

all of this questions without replacement.

(vi) Solve all the questions from i — v with replacement.

(i) ① | 6 | 5 | 4 |

$6 * 5 * 4 = 120$ ways.

② $P^6_3 = \frac{6!}{3!} = 6 * 4 * 5 = \boxed{120 \text{ ways}}$

(ii) الارقام اقل من 400 اي من 399 وهذا يعني لا يكون رقم المئات
يجب اما 3 او 2 . وبذلك فان

①

لانه لدينا رقمين اثنين فقط هما (2 و 3)

| 2 | 5 | 4 |

$2 * 5 * 4 = \boxed{40}$

② $P^2_1 \cdot P^5_2 = \frac{2!}{1!} \cdot \frac{5!}{3!} = 2 * 5 * 4 = \boxed{40 \text{ ways}}$

(iii) تكون الارقام الزوجية اذا كانت الاحاد رقم زوجي وعدد الارقام الزوجية هي اما (6 او 2)
اي اثنان فقط.

①

| 5 | 4 | 2 |

$5 * 4 * 2 = 40$

$P^5_2 \cdot P^2_1 = 40$

---

or $P^5_1 \cdot P^4_1 \cdot P^2_1 =$

(iv) الرقم ~~[illegible]~~
من مضاعفات الرقم 5

$\boxed{5 \mid 4 \mid 1}$

① $5 * 4 * 1 = \boxed{20 \text{ ways}}$

② $P^5_2 \cdot P^1_1 = 5 * 4 * 1 = \boxed{20 \text{ ways}}$

or $P^5_1 \cdot P^4_1 \cdot P^1_1 = \boxed{20 \text{ ways}}$

(v) odd.

① $\boxed{5 \mid 4 \mid 4}$

$5 * 4 * 4 = \boxed{80 \text{ ways}}$

② $P^5_2 \; P^4_1 = 5 * 4 * 4 = \boxed{80 \text{ ways}}$

or $P^5_1 \cdot P^4_1 \cdot P^4_1 = 5 * 4 * 4 = \boxed{80 \text{ ways}}$

(i) withreplacment.

① $\boxed{6 \mid 6 \mid 6}$

$6 * 6 * 6 = \boxed{216 \text{ ways}}$

② $n^r = 6^3 = 6 * 6 * 6 = \boxed{216 \text{ ways}}$

or. $P^6_1 \cdot P^6_1 \cdot P^6_1 = \boxed{216 \text{ ways}}$

(ii) with replacement.

① $\boxed{2 \mid 6 \mid 6}$

$2 * 6 * 6 = \boxed{72 \text{ ways}}$

② $n^r \cdot n^r = (2^1) \cdot (6)^2 = \boxed{72 \text{ ways}}$

$P^2_1 \cdot P^6_1 \cdot P^6_1 = \boxed{72 \text{ ways}}$

(iii) withreplacement.

① $\boxed{6 \mid 6 \mid 2}$

$6 * 6 * 2 = \boxed{72 \text{ ways}}$

② $n^r \cdot n^r = 6^2 \cdot 2^1 = 6 * 6 * 2 = \boxed{72 \text{ ways}}$

or $P^6_1 \cdot P^6_1 \cdot P^2_1 = \boxed{72 \text{ ways}}$

(iv) with replacement

① $\boxed{6 \mid 6 \mid 1}$

$6 * 6 * 1 = \boxed{36 \text{ ways}}$

② $n^r \cdot n^r = 6^2 \cdot 1^1 = 6 \times 6 \times 1 = \boxed{36 \text{ ways}}$

or $P^6_1 \cdot P^6_1 \cdot P^1_1 = \boxed{36 \text{ ways}}$

(v) with replacement. ① $\boxed{6 \mid 6 \mid 4}$

$6 * 6 * 4 = \boxed{144}$

② $P^6_1 \cdot P^6_1 \cdot P^4_1 = \boxed{144}$

$n^r \cdot n^r = (6^2) \cdot (4)^1 = 6 * 6 * 4 = \boxed{144}$

(2) واجب

Ex

How many 4- digit numbers can be formed from the digits (1, 2, 3, 4, 5) if?

① repititions are not allowed. (without repilation).

② repititions are allowed. ((with repilation))

③ if the number is odd and without repilations.

④ 〃 〃 〃 〃 with replecment.

⑤ if the number is even without repilation.

⑥ 〃 〃 〃 〃 〃 with replicment.

Answer

① without repilation

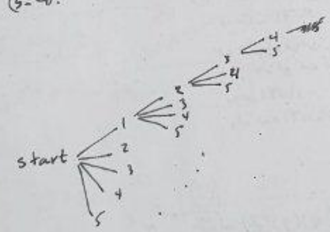
① Arrangment

5	4	3	2

$5 * 4 * 3 * 2 = \boxed{120}$ ways.

② $P^5_4 = \dfrac{5!}{(5-4)!} = 5! = 5*4*3*2*1 = \boxed{120 \text{ ways}}$

③



② with replecment

① Arrangment

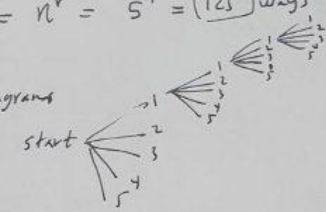
5	5	5	5

$5*5*5*5 = \boxed{125 \text{ ways}}$

② permutation

$$P = n^r = 5^4 = \boxed{125} \text{ ways}$$

③ tree Diagrams



③ if the number is odd without replecmet.

① Arrangment

4	3	2	3

$4*3*2*3 = \boxed{72 \text{ ways}}$

① نثبت الرقم الفردي
في خانة الاحاد.
لان الرقم يكون فردي
اذا كان او عدد فردي
وهو الاحاد.
ولان عدد الاعداد الفردية هو ③

② permut.

$$P^3_1 \, P^4_3 = \frac{3!}{2!} \cdot \frac{4!}{1!} = (3)(4)(3)(2) = \boxed{72 \text{ ways}}$$

الرقم الفردي
لانه ايشغل ذلا

③ Tree Diagrams H.w.

start.

4) if the number is odd with replecment.

① Arrangment

لنثبت الاعداد الفردية.

5	5	5	3

$5 \times 5 \times 5 \times 3 = \boxed{375 \text{ ways}}$

with repl.

without.

② Perm.

اثبت الزوجي ←

$P^3_1 \cdot P^5_1 \cdot P^5_1 \cdot P^5_1 = \frac{3!}{2!} \cdot \frac{5!}{4!} \cdot \frac{5!}{4!} \cdot \frac{5!}{4!} =$ ~~[illegible]~~

$= (3)(5)(5)(5)$

$= \boxed{375 \text{ ways}}$

or. $n^r \Rightarrow$ باستخدام تكرار $= (3)^1 \cdot (5)^3 = (3)(125) = 375$

عدد

$\cancel{P^3_1 \cdot n^{r-1}} = \boxed{\cancel{P^3_1 \cdot 5^3}}$

(5) even withoutreplecement

عدد 2,4,6,8 الزوجي ②

① Arrangment.

4	3	2	2

نثبت الزوجي اول

$4 \times 3 \times 2 \times 2 = \boxed{48 \text{ ways}}$.

② perm. $P^2_1 \cdot P^4_3 = \frac{(2!)}{1!} \cdot \frac{4!}{1!} = (2)(4)(3)(2) = \boxed{48 \text{ ways}}$

(6) even with replecment.

①

5	5	5	2

$\Rightarrow 5 \times 5 \times 5 \times 2 = \boxed{250}$ ways

$(n^r)(n^r) = (5)^3 \cdot (2)^1$

② Perm. $\cancel{P^2_1 \cdot P^5_1 \cdot P^5_1 \cdot P^5_1} = (2)(5)(5)(5) = \boxed{250}$

or $\cancel{P^2_1 \cdot n^{r-1} = P^2_1 (5^3)} = \boxed{250}$

H.w

(i) How many 4-digit number can be formed from the digit (1,4,5,8,7,6,2) a with and without replacement ?

(ii) the number that less than 5000 ?
(iii) the number is even ?
(iv) the number is odd. ?
(v) the number is multiple by 2 ?
(vi) the number is divided by 5 ?

with and without replacement

H.w

(i) Solve the problem if the first letters can not be Z ?
(ii) - - - second - - - y ?
(iii) - - - if the first digit can not be 9 ?
(iv) - - - - - Last - - - 8 ?

with and without replacement.

Example

(i) How may car numbers plate can be made if each plate contains 2-different letters followed by 3-different digits?

(ii) Solve the problem if the first digit can not be 0?

لدينا عدد الاحرف الانكليزية: $\boxed{26}$

a, ..., z.

لدينا عدد الارقام 0,1,2,3,4,
5,6,7,8,9

$= \boxed{10}$ ارقام (اعداد).

وبذلك كان الكل

(i) without replacement.

①

حرف	حرف	ارقام		
26	25	10	9	8

$= 26 * 25 * 10 * 9 * 8 = 468000$

② $P^{26}_2 \cdot P^{10}_3$ or → $P^{26}_1 \cdot P^{25}_1 \cdot P^{10}_1 \cdot P^{9}_1 \cdot P^{8}_1$

$$P^{26}_2 \cdot P^{10}_3 = \frac{26!}{24!} \cdot \frac{10!}{7!}$$

$$= \frac{26 * 25 * \cancel{24!}}{\cancel{24!}} \cdot \frac{10 * 9 * 8 * \cancel{7!}}{\cancel{7!}}$$

$$= 26 * 25 * 10 * 9 * 8$$

$$= \boxed{468000} \text{ ways.}$$

(ii)

0	1

نلاحظ بان 0 لا يمكن ان يكون في البداية

لانه مثلاً 035 فانه يقرأ 35. وبذلك كان صفر لا تأثير لاستخدامه.

في هذه الخانة نقصها. وبذلك كان الارقام.

Letters num.

26	25	9	9	8

عدا الصفر.

$26 * 25 * 9 * 9 * 8 = \boxed{421200}$ ways.

$$P^{26}_2 \cdot P^{9}_1 \cdot P^{9}_2 = \frac{26!}{24!} * \frac{9!}{8!} \cdot \frac{9!}{7!}$$

$$= 26 * 25 * 9 * 9 * 8$$

$$= \boxed{421200 \text{ ways}}$$

or $P^{26}_1 \cdot P^{25}_1 \cdot P^{9}_1 \cdot P^{9}_1 \cdot P^{8}_1$

(iii) use for (i) with replacement.

①

26	26	10	10	10

$26 * 26 * 10 * 10 * 10 =$

$$n^r \cdot n^r = (26)^2 \cdot (10)^3$$
$$=$$

(iv) use for (ii) with replacement.

①

26	26	9	10	10

$26 * 26 * 9 * 10 * 10 =$

②

$$n^r \cdot n^r \cdot n^r = (26)^2 \cdot (9) \cdot (10)^2$$
$$= (26)(26) \cdot (9)(10)(10)$$
$$=$$

(iv)

if the letter is not found in the first

25	25	10	9	8

$P^{25}_{1} \cdot P^{25}_{1} \cdot P^{10}_{3} =$

with

25	26	10	10	10

$=$

$(25)^1 (26)^1 (10)^3 =$

22 (i) In How many way can 3 boys a 2 girls sit in a row?

22/ ① $n! = 5! = 5 * 4 * 3 \times 2 * 1 = \boxed{120 \text{ ways}}$

② $P^5_5 = 5! = \boxed{120 \text{ ways}}$

③

5	4	3	2	1

$5 * 4 * 3 * 2 * 1 = \boxed{120 \text{ ways}}$

(ii) In how many way can they sit in a row if just the boys are to sit together.

هنا فقط الولد يجلسون معاً .
وهم عددهم (3) .
فنعتبرهم واحد .
ولدينا بنتان
فهذا يعني لدينا ثلاثة

①

3	2	1

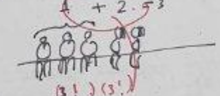
3	2	1

$3 * 2 * 1 * 3 * 2 * 1 = \boxed{36 \text{ ways}}$

② $3! \quad 3! = 3 * 2 * 1 * 3 * 2 * 1$
$= 36$ ways

P^3_3

1 + 2 = 3



(iii) How many can they sit in a row if just the girls are sit together.

4! = (3. + 1)

① $2! \cdot 4! = 2 * 1 * 4 * 3 * 2 * 1$
$= 2 * 24$
$= \boxed{48 \text{ ways}}$

2	1

4	3	2	1

$2 \times 1 * 4 \times 3 * 2 * 1 = \boxed{48 \text{ ways}}$

(iv) How many ways can they sit in a row if the boys and girls are each to sit together?

(v) In How many ways can they site in a row if just the girls are to site together?

(iv)

هناك طريقتان للتوزيع اما

GGBBB

او BBBGG

وفي كلا الحالتين فالاولاد لهم من الطرق

$\boxed{3|2|1}$

$3! = 6$

والبنات لهم

$\boxed{2|1}$

$2! = 2$

وبجمعهم معهم فسيكون لدينا

$3! \cdot 2!$

وهنا لدينا مجموعتين فنضربهم ب 2

فيكون الكل

بنات ولد * ولد بنات

$= 2!\,3! + 3!\,2!$

$= 2 \cdot 2!\,3!$

$= 2 \cdot 2 \cdot 6 = 24$

$\boxed{3! \cdot 2! \cdot 2}$

$6 * 2 * 2 = \boxed{24 \text{ ways}}$

or

$3!\;3! = 6 \times 6 = 36$

(v)

هناك اربع طرق لتوزيعهم حسب الترتيب اما

GGBBB

BGGBB

BBBGG

BBGGB

نلاحظ في كل الحالات فان البنات يجلسون مع بعضهم.

$2!$

والاولاد $3!$

ولدينا 4 مجموعات 4 مثل ذلك

$\boxed{2! \cdot 3! \cdot 4}$

$2 * 6 * 4 = \boxed{48 \text{ ways}}$

or

$= \boxed{2! \cdot 4!} = 48$

There are 6 roads between A and B and 4 Roads between B and C

(i) In How many way can one drive from A to C by way of B?

(ii) In How many ways can one drive from A to C and Back to A; Passing through B on both trips?

(iii) In How many ways can one drive the circular trip described in (ii) without using the same road more than once

(i)

1	2
6.	4

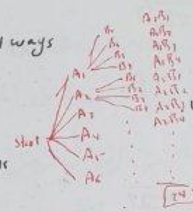
$6 * 4 = 24$

$A \xrightarrow{6} B$, $B \xrightarrow{4} C$

$P^6_1 \cdot P^4_1 = 6 * 4 = 24$ ways

start

$= 24$ ways



كل واحد: ايه ٤ خانة وصلت

(ii)

الذهاب		العودة	
6	4	4	6

$A \xrightarrow{6} B \xrightarrow{4} C$, $C \xrightarrow{4} B \xrightarrow{6} A$

① $6 * 4 * 4 * 6 = 576.$

② $P^6_1 \cdot P^4_1 \cdot P^4_1 \cdot P^6_1 = 6 * 4 * 4 * 6 = \boxed{576 \text{ ways}}$

or ③ $(6)^2 \cdot (4)^2$

with replacement

$n^r \cdot n^r = 6^2 \cdot 4^2$
$= 6 * 6 * 4 * 4$

(iii)

6	4	3	5

without replacement

① $6 * 4 * 3 * 5 = \boxed{360 \text{ ways}}$

or

6	5	4	3.

$P^6_2 \cdot P^4_2 = 6 * 5 * 4 * 3$

② $P^6_1 \cdot P^4_1 \cdot P^3_1 \cdot P^5_1 = 360$ ways

③ or $\boxed{P^6_2 \cdot P^4_2}$

(10)

In How many ways can 6-pearson seated.

(i) in a row (ii) only two pearson site side by side

(iii) only 3 pearson sit side by side. (iv) in a circular.

(v) only two pearso site side by side on a circulare.

(i) ① $n! = 6! = 6*5*4*3*2*1 = \boxed{720 \text{ ways}}$

②

6	5	4	3	2	1

$6*5*4*3*2*1 = \boxed{720 \text{ ways}}$

③ $P^6_6 = \frac{n!}{(n-n)!} = n! = 6! = \boxed{720 \text{ ways}}$

(ii) نعتبر الاثنين كشخص واحد.
فيصبح من 6 فقط 5 وبالتالي.

$2! \; 5! = (120)(2) = \boxed{240 \text{ ways}}$

(iii) $3! \; 4! = (24)(6) = \boxed{144 \text{ ways}}$

(iv) $(n-1)! = (5!) = \boxed{120 \text{ ways}}$.

(v) $2!(n-1)! = (2!)(5!) = (2)(120) = \boxed{240 \text{ ways}}$

2- site side by side | Circular

00 | دائرة

2! | $(n-1)!$ $n=6$

$(6-1)! = 5!$

$2!\,(n-1)!$

$\boxed{2! \; 5!}$