Ex7: In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Evaluate the following probabilities:

1. A person has type O blood.
2. A person has type A or type B blood.
3. A person has neither type A nor type O blood.
4. A person does not have type AB blood.

Sol:

1. The frequency of a person has type O

$$P(O) = \frac{n(O)}{n(S)} = \frac{21}{50}$$

2. Add the frequencies of the two classes

$$P(A\ or\ B) = P(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{n(A) + n(B)}{n(S)} = \frac{22 + 5}{50} = \frac{27}{50}$$

3. Neither A nor O means that a person has either type B or type AB blood

$$P(neither\ A\ nor\ O) = P(B\ or\ AB)$$

$$P(B \cup AB) = \frac{n(B \cup AB)}{n(S)} = \frac{n(B) + n(AB)}{n(S)} = \frac{5 + 2}{50} = \frac{7}{50}$$

4. To find the probability of not AB by subtracting the probability of type AB from 1.

$$P(not\ have\ type\ AB\ ) = P(AB^C) = 1 - P(AB) =$$

$$1 - \frac{n(AB)}{n(S)} = 1 - \frac{2}{50} = \frac{48}{50}$$

or

$$P(not\ have\ type\ AB\ ) = P(A\ or\ B\ or\ O) = \frac{22}{50} + \frac{5}{50} + \frac{21}{50} = \frac{48}{50}$$

Ex8: Hospital records indicated that maternity patients stayed in the hospital for the number of days shown in the distribution.

Number of days stayed	3	4	5	6	7
Frequency	15	32	56	19	5

Find the following probabilities.

1. A patient stayed exactly 5 days.

2. A patient stayed less than 6 days.

3. A patient stayed at most 4 days.

Sol:

1.

$$P(5\ dayes) = \frac{n(5\ days)}{n(S)} = \frac{56}{127} = 0.44$$

2. $P(less\ than\ 6\ dayes) = P(5\ dayes\ or\ 4\ dayes\ 3\ dayes) =$

$$P(less\ than\ 6\ dayes) = P(5\ dayes \cup 4\ dayes \cup 3\ dayes)$$

$$P(less\ than\ 6\ dayes) = \frac{n(5\ dayes \cup 4\ dayes \cup 3\ dayes)}{n(S)}$$

$$P(less\ than\ 6\ dayes) = \frac{15 + 32 + 56}{127} = \frac{103}{127} = 0.81$$

3. $(at\ most\ 4\ dayes) = P(3\ dayes\ or\ 4\ dayes)$

$$P(at\ most\ 4\ dayes) = P(3\ dayes \cup 4\ dayes)$$

$$P(at\ most\ 4\ dayes) = \frac{n(3\ dayes \cup 4\ dayes)}{n(S)} = \frac{n(3\ days)}{n(S)} + \frac{n(4\ days)}{n(S)}$$

$$P(at\ most\ 4\ dayes) = \frac{15}{127} + \frac{32}{127} = 0.12 + 0.25 = 0.37$$

Ex9: Two dice are thrown and the sum of the numbers which come up on the dice is noted. Let us consider the following events associated with this experiment

A: the sum is even.

B: the sum is a multiple of 3.

C: the sum is less than 4.

D: the sum is greater than 11.

Which pairs of these events are mutually exclusive?

Sol:

There are 36 elements in the sample space (see Ex5). Then:

$$A = \{(1,1),(1,3),(1,5),(2,2),(2,4),(2,6),(3,1),(3,3),(3,5),$$
$$(4,2),(4,4),(4,6),(5,1),(5,3),(5,5),(6,2),(6,4),(6,6)\}$$

$$B = \{(1,2),(2,1),(1,5),(5,1),(3,3),(2,4),$$
$$(4,2),(3,6),(6,3),(4,5),(5,4),(6,6)\}$$

$$C = \{(1,1),(2,1),(1,2)\}$$

$$D = \{(6,6)\}$$

$$A \cap B = \{(1,5),(2,4),(3,3),(4,2),(5,1),(6,6)\} \neq \emptyset$$

Therefore, A and B are not mutually exclusive events.

Similarly $A \cap C \neq \emptyset$, $A \cap D \neq \emptyset$, $B \cap C \neq \emptyset$, $B \cap D \neq \emptyset$, Thus, the pairs of events, (A, C), (A, D), (B, C), (B, D) are not mutually exclusive events.

But $C \cap D = \emptyset$, then C and D are mutually exclusive events.

Ex10: A coin is tossed three times, consider the following events. A: No head appears, B: Exactly one head appears and C: At least two heads appear. Do they form a set of mutually exclusive and exhaustive events?

Sol:

The sample space of the experiment is

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$,

$A = \{TTT\}, B = \{HTT, THT, TTH\}, C = \{HHT, HTH, THH, HHH\}$

Now

$A \cup B \cup C = \{TTT, HTT, THT, TTH, HHT, HTH, THH, HHH\} = S$

Therefore, A, B and C are exhaustive events.

Also, $A \cap B = \emptyset$, $A \cap C = \emptyset$ and $B \cap C = \emptyset$

Therefore, the events are pair-wise disjoint, i.e., they are mutually exclusive.

Hence, A, B and C form a set of mutually exclusive and exhaustive events.

Ex11: Let S be a sample space associated with the experiment 'examining three consecutive pens produced by a machine and classified as Good (non-defective) and bad (defective)^C. We may get $0, 1, 2, 3$ defective pens as result of this examination.

A sample space associated with this experiment is

$S = \{BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG\}$,

where B stands for a defective or bad pen and G for a non – defective or good pen.

Let:

A: there is exactly one defective pen,

B: there are at least two defective pens.

Hence $A = \{BGG, GBG, GGB\}$ and $B = \{BBG, BGB, GBB, BBB\}$

$$P(A) = \frac{n(A)}{n(S)} \text{ , } where\ A = \sum_{I=1}^{3} A_i = A_1 + A_2 + A_3$$

$$P(A) = \frac{n(A_1) + n(A_2) + n(A_3)}{n(S)} = \frac{3}{8}$$

$$P(B) = \frac{n(B)}{n(S)} \text{ , } where\ B = \sum_{I=1}^{4} B_i = B_1 + B_2 + B_3 + B_4$$

$$P(B) = \frac{n(B_1) + n(B_2) + n(B_3) + n(B_4)}{n(S)} = \frac{4}{8}$$

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B) = \frac{3}{8} + \frac{4}{8} = \frac{7}{8}$$

Ex12: Let us consider an experiment of tossing a coin three times. The sample space of experiment this is given as:

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Let $A = \{HHT, HTH, THH\}$ and $B = \{HTH, THH, HHH\}$ be two events, then

$A \cup B = \{HHT, HTH, THH, HHH\}$

$A \cap B = \{THH, HTH\}$

Now

$$P(A) = P(B) = \frac{3}{8} \ \ and \ \ P(A \cap B) = \frac{2}{8}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3}{8} + \frac{3}{8} - \frac{2}{8} = \frac{4}{8} = \frac{1}{2}$$

$$P(A^C) = 1 - P(A) = 1 - \frac{3}{8} = \frac{5}{8}$$

$$P(B^C) = 1 - P(B) = \frac{5}{8}$$

$$P(A \cap B^C) = P(A) - P(A \cap B) = \frac{3}{8} - \frac{2}{8} = \frac{1}{8}$$

$$P(A^C \cap B^C) = 1 - P(A \cup B) = 1 - \frac{4}{8} = \frac{4}{8} = \frac{1}{2}$$

$$P(A^C \cup B^C) = 1 - P(A \cap B) = 1 - \frac{2}{8} = \frac{6}{8} = \frac{3}{4}$$

Ex13: One card is drawn from a well shuffled deck of 52 cards. If each outcome, calculate the probability that the card will be

1. a diamond
2. not an ace
3. a black card, i.e., a club or, a spade
4. not a diamond
5. not a black card.

Sol:

When a card is drawn from a well shuffled deck of 52 cards, the number of possible outcomes is 52.

1. Let E_1 be the event that the card drawn is a diamond, Clearly the number of elements in set E_1 is 13, therefore,

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

2. We assume that the event of card drawn is an ace is E_2, therefore the card drawn is not an ace should be E_2^C, thus

   $$P(E_2^C) = 1 - P(E_2) = 1 - \frac{n(E_2)}{n(S)} = 1 - \frac{4}{52} = \frac{38}{52} = \frac{12}{13}$$

3. Let E_3 denote the event that the card drawn is black card, therefore, number of elements in the set $E_3 = 26$

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

4. We assumed in (1) above that E_1 is the event that the card drawn is a diamond, so the event that the card drawn is not a diamond is E_2^C, thus:

$$P(E_1^C) = 1 - P(E_1) = 1 - \frac{n(E_1)}{n(S)} = 1 - \frac{1}{4} = \frac{3}{4}$$

The event that the card drawn is not a black card may be denoted as is E_3^C, thus:

$$P(E_3^C) = 1 - P(E_3) = 1 - \frac{n(E_3)}{n(S)} = \frac{1}{2}$$

Ex14: A bag contains 9 discs of which 4 are red, 3 are blue and 2 are yellow. The discs are similar in shape and size. A disc is drawn at random from the bag. Calculate the following probabilities:

1. the disc is red.

2. the disc is yellow.

3. the disc is blue.

4. the disc is not blue.

5. the disc is either red or blue.

Sol:

There are 9 discs in all so the total number of possible outcomes is 9. Let the events A, B, C be defined as:

A: the disc drawn is red

B: the disc drawn is yellow

C: the disc drawn is blue.

1. The number of red discs = 4, i.e., $n\ (A) = 4$, Hence

$$P(A) = \frac{n(A)}{n(S)} = \frac{4}{9}$$

2. The number of yellow discs = 2, i.e., $n\ (B) = 2$, Hence

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{9}$$

3. The number of blue discs = 3, i.e., $n\ (C) = 2$, Hence

$$P(C) = \frac{n(C)}{n(S)} = \frac{3}{9} = \frac{1}{3}$$

4. Clearly the event not blue is ‘not C’. We know that

$$P(not\ C) = 1 - P(C)\ or\ P(C^C) = 1 - P(C)$$

$$P(not\ C) = 1 - \frac{1}{3} = \frac{2}{3}$$

5. The event either red or blue is described by the set $(A\ or\ C)$. Since, $A\ and\ C$ are mutually exclusive events, we have

$$P(A\ or\ C) = P\ (A\ \cup\ C) = P(A) + P(C) = \frac{4}{9} + \frac{3}{9} = \frac{7}{9}$$

Ex15: Two students Rami and Susan appeared in an examination. The probability that Rami will fail the examination is 0.05 and that Susan will fail the examination is 0.10. The probability that both will fail the examination is 0.02. Find the probability that:

1. Both Rami and Susan will not fail the examination.

2. At least one of them will not fail the examination.

3. Only one of them will fail the examination.

Sol: Let E and F denote the events that Rami and Susan will fail the examination, respectively. Given that

$$P(E) = 0.05, P(F) = 0.10, P(E \cap F) = 0.02$$

1. The event both Rami and Susan will not fail the examination is expressed as $E^C \cup F^C$. Since, E^C is (not E), i.e., Rami will not fail the examination and F^C is (not F), i.e., Susan will not fail the examination.

   Also $E^C \cup F^C = (E \cap F)^C$ (by Demorgan's Law)

   Now $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

   $P(E \cup F) = 0.05 + 0.10 - 0.02 = 0.13$

   $P(E^C \cup F^C) = P(E \cap F)^C = 1 - P(E \cup F) = 1 - 0.13 = 0.87$

2. P (at least one of them will not fail) =1 – P (both of them will fail)

   P (at least one of them will not fail) =1 – 0.02 = 0.98

3. The event only one of them will fail the examination is same as the event either (Rami will fail, and Susan will not fail) or (Rami will not fail and Susan will fail) i.e., $E \cap F^C$ or $E^C \cap F$, where ., $E \cap F^C$ and $E^C \cap F$ are mutually exclusive. Therefore,

   $P(only\ one\ of\ them\ will\ fail) = P(E \cap F^C\ or\ E^C \cap F)$

   $P[(E \cap F^C) \cup (E^C \cap F)] = P(E \cap F^C) + P(E^C \cap F)$

   $P[(E \cap F^C) \cup (E^C \cap F)] = P(E) - P(E \cap F) + P(F) - P(E \cap F)$

   $P[(E \cap F^C) \cup (E^C \cap F)] = 0.05 - 0.02 + 0.10 - 0.02 = 0.11$