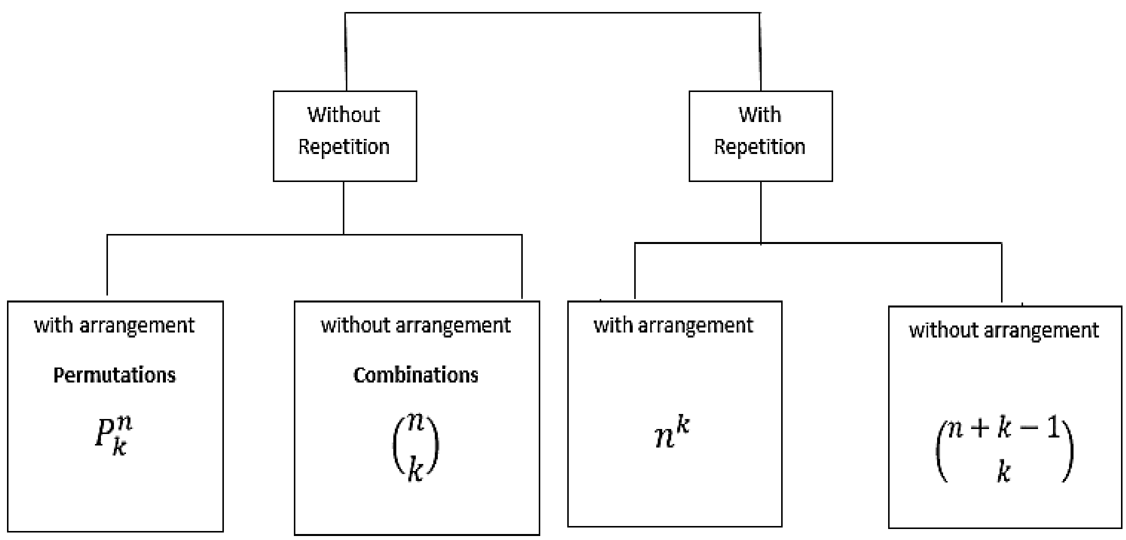
Chapter Two

Counting Methods

2.1 Introduction:

The methods of selecting (k) items from among (n) items differ according to the conditions of this event, the choice may be made with or without repetition, and the arrangement condition may be imposed on that choice. The diagram below represents the formulas used to find the number of possible ways to select (k) items from among (n) items:



2.1.1 Combinations:

It is the process of selecting (k) items from a population of (n) items without repetition and regardless of the arrangement, or it is the number of possible ways to choose (k) items from a population of (n) items. The law of compatibility can be defined mathematically according to the following formula:

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} \quad , k \leq n$$

Ex16: In how many ways can a sample of size 3 be selected from a population of 4?

Sol: $n = 4, k = 3$

$$n(E) = \binom{4}{3} = \frac{4!}{3!\,(4-3)!} = \frac{4 \times 3!}{3! \quad 1!} = 4$$

Ex17: It is intended to choose a committee consisting of 3 peoples from among 4 men and 4 women. What is the possible ways to choose this committee? and what is the possible ways to choose each of the following:

1. This committee will be composed of men only.
2. This committee consists of two men and a woman.
3. This committee includes at least one man.

Then find the following probabilities:

4. This committee will be composed of men only.
5. This committee consists of two men and a woman.
6. This committee includes at least one man.

Sol:

$$n(S) = \binom{8}{3} = \frac{8!}{3!\,(8-3)!} = \frac{8 \times 7 \times 6 \times 5!}{3 \times 2 \times 1 \quad 5!} = 56$$

1. The committee includes men only

$$n(E_1) = \binom{4}{3} = \frac{4!}{3!\,(4-3)!} = \frac{4 \times 3!}{3!\ \ 1!} = 4$$

2. The committee consists of two men and a woman

$$n(E_2) = \binom{4}{2}\binom{4}{1} = \frac{4!}{2!\,(4-2)!}\ \frac{4!}{1!\,(4-1)!} = \frac{4 \times 3!}{1\ \ 3!} = 24$$

3. The committee includes at least one man

$$n(E_3) = \binom{4}{1}\binom{4}{2} + \binom{4}{2}\binom{4}{1} + \binom{4}{3}\binom{4}{0}$$

$$n(E_3) = \frac{4!}{1!\,(4-1)!}\ \frac{4!}{2!\,(4-2)!} + \frac{4!}{2!\,(4-2)!}\ \frac{4!}{1!\,(4-1)!} + \frac{4!}{3!\,(4-3)!}\ \frac{4!}{0!\,(4-0)!}$$

$$n(E_3) = \frac{4 \times 3!}{1!\ \ 3!}\ \frac{4 \times 3 \times 2!}{2 \times 1\ \ 2!} + \frac{4 \times 3 \times 2!}{2 \times 1\ \ 2!}\ \frac{4 \times 3!}{1!\ \ 3!} + \frac{4 \times 3!}{3!\ \ 1!}$$

$$n(E_3) = 24 + 24 + 4 = 52$$

4. Probability that the committee will be composed of men only

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{56} = 0.07$$

5. Probability that the committee consists of two men and a woman

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{24}{56} = 0.43$$

6. Probability that the committee includes at least one man

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{52}{56} = 0.93$$

Ex18: A box contains six black balls and four red balls, calculate the following probabilities

1. Get three black balls
2. Get two red balls

Sol:

$$nS = \binom{10}{3} = \frac{10!}{3!\,(10-3)!} = \frac{10 \times 9 \times 8 \times 7!}{3 \times 2 \times 1 \;\; 7!} = 120$$

1. three black balls

$$n(E_1) = \binom{6}{3} = \frac{6!}{3!\,(6-3)!} = \frac{6 \times 5 \times 4 \times 3!}{3 \times 2 \times 1 \;\; 3!} = 20$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{20}{120} = 0.17$$

2. two red balls

$$nE_2 = \binom{4}{2} = \frac{4!}{2!\,(4-3)!} = \frac{4 \times 3 \times 2!}{2 \times 1 \;\; 3!} = 6$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{120} = 0.017$$

Ex19: A box contains six black balls and four red balls, assuming that three balls are drawn without return, find each of the following possibilities:

1. Get two red balls
2. Get three balls of the same color
3. Get at least one red ball

Sol:

$$n(S) = \binom{10}{3} = \frac{10!}{3!\,(10-3)!} = \frac{10 \times 9 \times 8 \times 7!}{3 \times 2 \times 1 \quad 7!} = 120$$

1. two red balls

$$n(E_1) = \binom{4}{2}\binom{6}{1} = \frac{4!}{2!\,(4-2)!}\ \frac{6!}{1!\,(6-1)!} = \frac{4 \times 3 \times 2!}{2 \times 1 \quad 2!}\ \frac{4 \times 5!}{1 \quad 5!} = 24$$

$$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{24}{120} = 0.2$$

2. three balls of the same color

$$n(E_2) = \binom{6}{3} + \binom{4}{3} = \frac{6!}{3!\,(6-3)!} + \frac{4!}{2!\,(4-3)!}$$

$$n(E_2) = \frac{6 \times 5 \times 4 \times 3!}{2 \times 1 \quad 3!} + \frac{4 \times 3 \times 2!}{2 \times 1 \quad 2!} = 26$$

$$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{23}{120} = 0.22$$

3. three balls of the same color

$$n(E_3) = \binom{6}{2}\binom{4}{1} + \binom{6}{1}\binom{4}{2} + \binom{6}{0}\binom{4}{3}$$

$$n(E_3) = \frac{6!}{2!\,(6-2)!}\ \frac{4!}{1!\,(4-1)!} + \frac{6!}{1!\,(6-1)!}\ \frac{4!}{2!\,(4-2)!} + \frac{6!}{0!\,(6-0)!}\ \frac{4!}{3!\,(4-3)!}$$

$$n(E_3) = \frac{6 \times 5 \times 4!}{2 \quad 4!}\ \frac{4 \times 3!}{1 \quad 3!} + \frac{6 \times 5!}{1 \quad 5!}\ \frac{4 \times 3 \times 2!}{2 \quad 2!} + \frac{4 \times 3!}{3! \quad 1!}$$

$$n(E_3) = 60 + 36 + 4 = 100$$

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{100}{120} = 0.83$$