2.1.2 Permutations and other counting:

Permutations are defined as the number of possible arrangements for k elements of event E taken from a population of size n, permutations are used in calculating the number of ways for a set of elements provided repetition is not allowed and the arrangement is taken into account. Permutations can be expressed mathematically according to the following law:

$$P_k^n = \frac{n!}{(n-k)!}$$

Ex20: If the arrangement is taken into account, what is the number of possible ways to choose three letters among the letters a, b, c, d, provided that the same letter is not repeated in the same arrangement.

or

What is the number of possible arrangements to choose three letters from among the letters a, b, c, d, provided that the same letter is not repeated in the same arrangement?

Sol:

$$n(E) = P_3^4 = \frac{4!}{(4-3)!} = 4! = 24$$

$abc, acb, bac, bca, cab, cba$

$abd, adb, bad, bda, dab, dba$

$acd, adc, cad, cda, dac, dca$

$bcd, bdc, cbd, cdb, dbc, dcb$

Ex21: How many different three-letter words can be formed from the letters of the word Erbil?

Sol:

$$n(E) = P_3^5 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60$$

Ex22: In how many ways can five students sit in a row of eight chairs?

Sol:

$$n(E) = P_5^8 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = 6720$$

Ex23: What is the number of possible arrangements to choose three letters from among the letters a, b, c, d, note that repetition and arrangement are allowed?

Sol: $n(E) = n^k = 4^3 = 64$

Ex24: How many different three-letter words can be formed from the letters of the word (Erbil) if repetition and arrangement are allowed?

Sol:

$$n(E) = n^k = 5^3 = 125$$

Ex25: A box contains 12 red balls and 8 white balls, if 3 red balls and 2 black balls are drawn, in how many ways can those balls be drawn if repetition and arrangement are allowed?

Sol:

$$n(E) = n^r\, m^b = 12^3\, 8^2 = 101592$$

Ex26: A box contains 12 red balls and 8 white balls, if 3 red balls and 2 black balls are drawn, in how many ways can those balls be drawn if repetition not allowed and arrangement is allowed?

Sol:

$$n(E) = P_r^n \, P_b^m = P_3^{12} \, P_2^8 = \frac{12!}{(12-3)!} \, \frac{8!}{(8-2)!} = 73920$$

Ex27: A box contains 12 red balls and 8 white balls, if 3 red balls and 2 black balls are drawn, in how many ways can those balls be drawn without repetition and arrangement?

Sol:

$$n(E) = \binom{n}{r} \binom{m}{b} = \binom{12}{3} \binom{8}{2} = \frac{12!}{3! \, (12-3)!} \, \frac{8!}{2! \, (8-2)!} = 6160$$

Ex28: A box contains 6 red balls and 4 white balls, if 4 balls are drawn. Find the probability of getting 3 red balls and 1 black ball if repetition and arrangement are allowed

Sol:

$$n(S) = n^k = 10^4 = 10000$$

$$n(E) = n^r \, m^b = 6^3 \, 4^1 = 864$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{846}{10000} = 0.09$$

Ex29: A box contains 6 red balls and 4 white balls, if 4 balls are drawn. Find the probability of getting 3 red balls and 1 black ball if repetition is not allowed and arrangement is allowed.

Sol:

$$n(S) = P_k^n = P_4^{10} = \frac{10!}{(10-4)!} = 5040$$

$$n(E) = P_r^n \, P_b^m = P_3^6 \, P_1^4 = \frac{6!}{(6-3)!} \, \frac{4!}{(4-2)!} = 1440$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{1440}{5040} = 0.29$$

Ex30: A box contains 6 red balls and 4 white balls, if 4 balls are drawn. Find the probability of getting 3 red balls and 1 black ball if repetition and arrangement are not allowed

Sol:

$$n(S) = \binom{n}{k} = \binom{10}{4} = \frac{10!}{4! \, (10-4)!} = 210$$

$$n(E) = \binom{n}{r} \binom{m}{b} = \binom{6}{3} \binom{4}{1} = \frac{6!}{3! \, (6-3)!} \, \frac{4!}{2! \, (4-2)!} = 120$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{120}{210} = 0.57$$