Random Experiment:

An experiment is an act or procedure or phenomenon which have different results when it is repeated under identical conditions. The experiment that we can not control of it outcomes is called "Random Experiment". In other word an experiment is called random experiment if it satisfy the following conditions

1- It has more than one possible outcome.
2- It is not possible to predicting its outcomes.

Event: An event is a subset of the sample space. Eevents can be classified into several types: sure events, impossible events, complement events, mutually exclusive events and independent events.

Probability:

Probability is a way of summarizing the uncertainty of events, it gives a numerical measure for certainty or uncertainty.
Another way to define probability, it is the ratio of event outcomes to the total number of outcomes. Suppose that S is a sample space and E is an event, then $E \subset S$ and:

$$\text{Probability of event}(E) = \frac{\text{number of event outcomes}}{\text{total number of outcomes}}$$

which denoted mathematicaly as

$$P_r(E) = \frac{n(E)}{n(S)}$$

Basic Concepts of Probability

[1] Let E is any event in the sample space, then
$0 \leq Pr(E) \leq 1$

[2] If any event E cannot appear, then $Pr(E) = 0$

[3] Let $E_1, E_2, \ldots, E_m$ are events in S, then

$$\sum_{j=1}^{m} Pr(E_j) = 1$$

[4] $Pr(\phi) = 0$

Types of events: Eevents can classified into:

[1] sure event: An event E is called sure event iff $Pr(E) = 1$

[2] Impossible event: An event E is said to be impossible iff $Pr(E) = 0$

[3] Simple event: If an event E has one sample point, then it is called simple event. For example tossing a coin or rolling a die,

[4] Compound event: If an event E has more than one point in the Sample space then it is called compound event, for example an experiment of tossing three coins or throwing 2 dice.

[5] Complement event: E is a set of outcomes, $E \in S$, then the complement event of E is denoted by E^c or $\bar{E}$ is a set of outcomes that are not in E, $E^c \in S$, $E^c \notin E$.

EX1: When we flip a coin, then the sample space is $S = \{H, T\} \Rightarrow n(S) = 2$
where H: Head and T: Tail, then

$$Pr(H) = \frac{n(H)}{n(S)} = \frac{1}{2} \text{ and } Pr(T) = \frac{n(T)}{n(H)} = \frac{1}{2}$$

EX2: Let a die is roll, then write the sample space, and find the probability for each of the following events:

E_1: Get any number.
E_2: Get an odd number.
E_3: Get an even number.
E_4: Get a number less than 5.
E_5: Get a number greater than 3.
E_6: Get a prime number.
E7: Get a digit number.

Sol: $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

- E_1: the number maybe 1, 2, 3, 4, 5 and 6
so $n(E_1) = 1$ (sure event)
$$Pr(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{6}$$

- $E_2 = \{1, 3, 5\} \Rightarrow n(E_2) = 3$
$$Pr(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

- $E_3 = \{2, 4, 6\} \Rightarrow n(E_2) = 3$
$$Pr(E_3) = \frac{n(E_3)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

- $E_4 = \{1, 2, 3, 4\} \Rightarrow n(E_4) = 4$
$$Pr(E_4) = \frac{n(E_4)}{n(S)} = \frac{4}{6} = \frac{2}{3}$$

- $E_5 = \{1, 2\} \Rightarrow n(E_5) = 2 \Rightarrow Pr(E_5) = \frac{n(E_5)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

- $E_6 = \{1, 2, 3, 5\} \Rightarrow n(E_6) = 4 \Rightarrow Pr(E_6) = \frac{n(E_6)}{n(S)} = \frac{4}{6} = \frac{2}{3}$

- $E_7 = \{1, 2, 3, 4, 5, 6\} = S \Rightarrow n(E_7) = 6$

  $Pr(E_7) = \frac{n(E_7)}{n(S)} = \frac{6}{6} = 1$

EX3: Two coins are tossed togther, write the Sample space and find the probability of the following events:

E_1: one Head occur.
E_2: Two Head occur.
E_3: Three Head occur.
E_4: at least ~~ones~~ ~~Head~~ appear.
E_5: a Head appears.
E_6: no Head appears

SOL: $S = \{(H, H), (H, T), (T, H), (T, T)\}$ or

$S = \{HH, HT, TH, TT\} \Rightarrow n(S) = n^{x} = 2^2 = 4$ (x: coins no., n: out no.)

- $E_1 = \{HT, TH\} \Rightarrow n(E_1) = 2 \Rightarrow Pr(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

- $E_2 = \{HH\} \Rightarrow n(E_2) = 1 \Rightarrow Pr(E_2) = \frac{n(E_2)}{n(S)} = \frac{1}{4}$

- $E_3 = \{\ \} = \phi \Rightarrow Pr(E_3) = 0$ (impossible event)

- $E_4 = \{HT, TH, HH\} \Rightarrow n(E_4) = 3 \Rightarrow Pr(E_4) = \frac{n(E_4)}{n(S)} = \frac{3}{4}$

- $E_5 = \{HT, TH, HH\} = E_4 \Rightarrow pr(E_5) = \frac{3}{4}$

- $E_6 = \{TT\} \Rightarrow n(E_6) = 1 \Rightarrow Pr(E_6) = \frac{n(E_6)}{n(S)} = \frac{1}{4}$

EX4: Three coins are tossing together. then find the following probabilities:

- E_1: appearance of one Head.
- E_2: appearance two Heads.
- E_3: three Heads occur.
- E_4: at least two Heads occur.
- E_5: at most two Head occur.
- E_6: no Head occurs.

SoL: $S = \{\underbrace{HHH}_{3H}, \underbrace{HHT, HTH, THH}_{2H}, \underbrace{HTT, THT, TTH}_{1H}, \underbrace{TTT}_{0H}\}$

$n(S) = n^x = 2^3 = 8$ (3 → coins, 2 → out)

- $E_1 = \{HTT, THT, TTH\} \Rightarrow n(E_1) = 3$

$Pr(E_1) = \dfrac{n(E_1)}{n(S)} = \dfrac{3}{8}$

- $E_2 = \{HHT, HTH, THH\} \Rightarrow n(E_2) = 3$

$Pr(E_2) = \dfrac{n(E_2)}{n(S)} = \dfrac{3}{8}$

- $E_3 = \{HHH\} \Rightarrow n(E_3) = 1 \Rightarrow Pr(E_3) = \dfrac{n(E_3)}{n(S)} = \dfrac{1}{8}$

- $E_4 = \{\underbrace{HHT, HTH, THH}_{2H}, \underbrace{HHH}_{3H}\} \Rightarrow n(E_4) = 4$

$Pr(E_4) = \dfrac{n(E_4)}{n(S)} = \dfrac{4}{8} = \dfrac{1}{2}$

- $E_5 = \{\underbrace{HTT, THT, TTH}_{1H}, \underbrace{HHT, HTH, THH}_{2H}\} \Rightarrow n(E_5) = 6$

$Pr(E_5) = \dfrac{n(E_5)}{n(S)} = \dfrac{6}{8} = \dfrac{3}{4}$

- $E_6 = \{TTT\} \Rightarrow n(E_6) = 1 \Rightarrow Pr(E_6) = \dfrac{n(E_6)}{n(S)} = \dfrac{1}{8}$

EX5: Two dice are thrown together, then find the following probabilities:

A- The appears number is 1.
B- The appears number is either 1 or 4.
C- The appears numbers are 1 and 4.
D- The sum of the two numbers is 6.
E- The sum of the two numbers is greater than 10.
F- The sum of the two numbers is less than 13.
G- The sum of the two numbers is 1.

SoL: $S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$
$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),$
$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$
$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$
$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

$n(S) = n^{x}_{out} = 6^{2 \to \text{dice}} = 36$

A- $A = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(3,1),(4,1),(5,1),(6,1)\} \Rightarrow$

$n(A) = 11$

$Pr(A) = \frac{n(A)}{n(S)} = \frac{11}{36}$

B- $B = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$
$(2,1),(3,1),(4,1),(5,1),(6,1),$
$(4,2),(4,3),(4,4),(4,5),(4,6),$
$(2,4),(3,4),(5,4),(6,4)\}$

$n(B) = 20$

$Pr(B) = \frac{n(B)}{n(S)} = \frac{20}{36}$

C. $C = \{(1,4),(4,1)\} \Rightarrow n(C) = 2 \Rightarrow Pr(C) = \frac{n(C)}{n(S)} = \frac{2}{36}$

D. $D = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$

$n(D) = 5$

$Pr(D) = \frac{n(D)}{n(S)} = \frac{5}{36}$

E. $E = \{(5,6),(6,5),(6,6)\} \Rightarrow n(E) = 3$

$Pr(E) = \frac{n(E)}{n(S)} = \frac{3}{36}$

F. F = all outcomes of sample space then

$Pr(F) = 1$ certain event

G. $G = \{\ \}$ or $\phi \Rightarrow Pr(\phi) = 0$ impossible event.

(p. 83)

EX6: A bag contains 8 white, 6 red balls, 4 balls ~~are~~ drawn ~~randomly~~ together. Find the probabilities of the following events:

A: There will be 2 red balls.
B: There will be at least 2 white balls.
C: There will be balls from the same color.
Hw D: There will be balls from different colors.

Sol: $n = 14$, $x = 4$, $n_1 = 8$, $n_2 = 6$

$$n(S) = C_x^n = \frac{n!}{x!(n-x)!}$$

$$n(S) = C_4^{14} = \frac{14!}{4!\,10!} = \frac{14*13*12*11*10!}{4*3*2*1\quad 10!} = 1001$$

A. $n(A) = n(2R \text{ and } 2W) = n(2R) \cdot n(2W)$

$n(A) = C_2^6 \cdot C_2^8 = \frac{6!}{2!\,4!} \cdot \frac{8!}{2!\,6!} = 15 * 28 = 420$

$$Pr(A) = \frac{n(A)}{n(S)} = \frac{420}{1001} = \frac{60}{143}$$

B. $n(B) = n(2W \text{ or } 3W \text{ or } 4W)$

$$n(B) = n(2W \cdot 2R) + n(3W \cdot 1R) + n(4W \cdot 0R)$$

$$n(B) = C^8_2 \cdot C^6_2 + C^8_3 \cdot \underbrace{C^6_1}_{6} + C^8_4 \cdot C^6_0$$

$$n(B) = \frac{8!}{2!\,6!} \cdot \frac{6!}{2!\,4!} + \frac{8!}{3!\,5!} \cdot \frac{6!}{1!\,5!} + \frac{8!}{4!\,4!}$$

$$n(B) = 420 + (56 * 6) + 70 = 826$$

$$Pr(B) = \frac{n(B)}{n(S)} = \frac{826}{1001} = \frac{118}{143}$$

Ex6: Re-Solve Ex5 if 4 balls are drawn respectively with replacement

Ex7: Re-Solve Ex5 if the balls are numbered and 4 ball are drawn respectively without replacement.

EX8: 20 g 25 b 3 selected (1) Boss and Vice

(2) the Boss is girl

(1) $P^{20}_1 \; P^{25}_1 \; P^{43}_1$

(2) $P^{20}_1 \; P^{44}_1 \; P^{43}_1$

Some properties of probability

- Let A and B are two events in S, then:

[1] $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$
$Pr(A \cap B) = Pr(A) + Pr(B) - Pr(A \cup B)$

[2] $Pr(A) + Pr(A^c) = 1 \Rightarrow Pr(A^c) = 1 - Pr(A)$
$Pr(B) + Pr(B^c) = 1 \Rightarrow Pr(B^c) = 1 - Pr(B)$

[3] $Pr(A \cap B)^c + Pr(A \cap B) = 1 \Rightarrow Pr(A \cap B)^c = 1 - Pr(A \cap B)$
$Pr(A \cup B)^c + Pr(A \cup B) = 1 \Rightarrow Pr(A \cup B)^c = 1 - Pr(A \cup B)$

[4] $Pr(A^c \cap B^c) = 1 - Pr(A \cup B)$, $Pr(A^c \cup B^c) = 1 - Pr(A \cap B)$

[5] $Pr(A \cap B^c) = Pr(A) - Pr(A \cap B)$
$Pr(A^c \cap B) = Pr(B) - Pr(A \cap B)$

- Let A, B and C are three events in S, then

[1] $Pr(A \cup B \cup C) = Pr(A) + Pr(B) + Pr(C) - Pr(A \cap B)$
$- Pr(A \cap C) - Pr(B \cap C) + Pr(A \cap B \cap C)$

[2] $Pr(A \cup B) \cap C = Pr(A \cap C) \cup Pr(B \cap C)$
$Pr(A \cap B) \cup C = Pr(A \cup C) \cap Pr(B \cup C)$

Ex9: For digits set and the following events:

A: even numbers.

B: numbers less than 3. D: odd numbers.

C: numbers greater than 3.

then calculate:

1- $Pr(A)$, $Pr(B)$, $Pr(C)$, $Pr(D)$

2- $Pr(A\cap B)$, $Pr(A\cap C)$, $Pr(B\cap C)$, $Pr(C\cap D)$

3- $Pr(A\cup B)$, $Pr(A\cup C)$, $Pr(B\cup C)$, $Pr(C\cup D)$

4- $Pr(A\cup B\cup C)$, $Pr(B\cup C\cup D)$

Sol:

$S=\{0,1,2,3,4,5,6,7,8,9\} \Rightarrow n(S)=10$

$A=\{2,4,6,8\} \Rightarrow n(A)=4$

$B=\{0,1,2\} \Rightarrow n(B)=3$

$C=\{4,5,6,7,8,9\} \Rightarrow n(C)=6$

$D=\{1,3,5,7,9\} \Rightarrow n(D)=5$

1. $Pr(A)=\dfrac{n(A)}{n(S)}=\dfrac{4}{10}$ $\quad Pr(B)=\dfrac{n(B)}{n(S)}=\dfrac{3}{10}$

$Pr(C)=\dfrac{n(C)}{n(S)}=\dfrac{6}{10}$ $\quad Pr(D)=\dfrac{n(D)}{n(S)}=\dfrac{5}{10}$.

2. $A\cap B=\{2\} \Rightarrow n(A\cap B)=1 \Rightarrow Pr(A\cap B)=\dfrac{n(A\cap B)}{n(S)}=\dfrac{1}{10}$

$A\cap C=\{4,6,8\} \Rightarrow n(A\cap C)=3 \Rightarrow Pr(A\cap C)=\dfrac{n(A\cap C)}{n(S)}=\dfrac{3}{10}$

$B\cap C=\{\ \}$ or $\phi \Rightarrow n(\phi)=0 \Rightarrow Pr(B\cap C)=Pr(\phi)=0$

$C\cap D=\{5,7,9\} \Rightarrow n(C\cap D)=3 \Rightarrow Pr(C\cap D)=\dfrac{n(C\cap D)}{n(S)}=\dfrac{3}{10}$

3. $Pr(A\cup B)=Pr(A)+Pr(B)-Pr(A\cap B)=\dfrac{4}{10}+\dfrac{3}{10}-\dfrac{1}{10}=\dfrac{6}{10}$

or $A \cup B = \{x : x \in A \text{ or } x \in B\} = \{0, 1, 2, 4, 6, 8\}$

$n(A \cup B) = 6 \Rightarrow Pr(A \cup B) = \frac{n(A \cup B)}{n(S)} = \frac{6}{10}$

$$Pr(A \cup C) = Pr(A) + Pr(C) - Pr(A \cap C)$$
$$= \frac{4}{10} + \frac{6}{10} - \frac{3}{10} = \frac{7}{10}$$

$$Pr(B \cup C) = Pr(B) + Pr(C) - Pr(B \cap C)$$
$$\frac{3}{10} + \frac{6}{10} - 0 = \frac{9}{10}$$

$$Pr(C \cup D) = Pr(C) + Pr(D) - Pr(C \cap D)$$
$$= \frac{6}{10} + \frac{5}{10} - \frac{3}{10} = \frac{8}{10}$$

4- $Pr(A \cup B \cup C) = Pr(A) + Pr(B) + Pr(C) - Pr(A \cap B)$
$- Pr(A \cap C) - Pr(B \cap C) + Pr(A \cap B \cap C)$

$A \cap B \cap C = \{x : x \in A, x \in B \text{ and } x \in C\} = \phi \Rightarrow Pr(\phi) = 0$

$\therefore Pr(A \cup B \cup C) = \frac{4}{10} + \frac{3}{10} + \frac{6}{10} - \frac{1}{10} - \frac{3}{10} - 0 - 0 = \frac{9}{10}$

$Pr(B \cup C \cup D) = Pr(B) + Pr(C) + Pr(D) - Pr(B \cap C) - Pr(C \cap D)$
$- Pr(B \cap D) + Pr(B \cap C \cap D)$

$B \cap D = \{x : x \in B \text{ and } x \in D\} = \{1\} \Rightarrow n(B \cap D) = 1$

$Pr(B \cap D) = \frac{n(B \cap D)}{n(S)} = \frac{1}{10}$

$B \cap C \cap D = \{x : x \in B, x \in C \text{ and } x \in D\} = \phi \Rightarrow Pr(A \cap B \cap C) = 0$

$Pr(B \cup C \cup D) = \frac{3}{10} + \frac{6}{10} + \frac{5}{10} - 0 - \frac{3}{10} - \frac{1}{10} - 0 = \frac{10}{10} = 1$

or

$B \cup C \cup D = \{x : x \in B \text{ or } x \in C \text{ or } x \in D\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$\therefore Pr(B \cup C \cup D) = \frac{n(B \cup C \cup D)}{n(S)} = \frac{10}{10} = 1$ $(n = 10)$

EX10: In a sample of 50 persons, 21 have type O blood. 22 have type A blood, 5 have type B blood and 2 have type AB blood. If a person selected at random, then evaluate the following probabilities:

1- A person has type O blood
2- A person has either type A or type B.
3- A person has niether type A nor type O.
4- A person does not has type AB blood.

SoL: $n(S) = 50$

1- $Pr(O) = \dfrac{n(O)}{n(S)} = \dfrac{21}{50}$

2- $Pr(A \text{ or } B) = Pr(A \cup B) = \dfrac{n(A \cup B)}{n(S)} = \dfrac{22 + 5}{50} = \dfrac{27}{50}$

or $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B) \rightarrow \text{Zero}$

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$Pr(A \cup B) = \dfrac{22}{50} + \dfrac{5}{50} - 0 = \dfrac{27}{50}$

3- $Pr\{not(A \text{ or } O)\} = Pr(A \cup O)^c = 1 - Pr(A \cup O)$
$= 1 - \dfrac{21 + 22}{50} = 1 - \dfrac{43}{50} = \dfrac{7}{50}$

or $Pr(\text{not } A \text{ or } O) = Pr(B \text{ or } AB) = \dfrac{n(B \cup AB)}{n(S)}$

$= \dfrac{5 + 2}{50} = \dfrac{7}{50}$

4- $Pr(\text{not } AB) = Pr(AB^c) = 1 - Pr(AB) = 1 - \dfrac{n(AB)}{n(S)}$
$= 1 - \dfrac{2}{50} = \dfrac{48}{50}$

or $(\text{not } AB) = (O \text{ or } A \text{ or } B) = \dfrac{21 + 22 + 5}{50} = \dfrac{48}{50}$.

Ex11: A coin is tossed and a die is rolled, the find probability of the following:

1. E_1: an even number occur.

   E_2: a head occur. (simple event)

   E_3: a head and a number less than 3. (compound event)

   E_4: get a tail and at least 3
2. E_1^c, $E_1 \cap E_2$, $E_1 \cap E_3$, $E_1 \cap E_4^c$, $E_1 \cap E_2^c$, $(E_1 \cap E_3)^c$, $(E_1 \cap E_2)^c$, $E_1 \cup E_2$

Sol: $S = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),$
$(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)\}$

$n(S) = 12$

1.

$E_1 = \{(H,2),(H,4),(H,6),(T,2),(T,4),(T,6)\} \Rightarrow n(E_1) = 6$

$$Pr(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{12}$$

$E_2 = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6)\} \Rightarrow n(E_2) = 6$

$$Pr(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{12}$$

$E_3 = \{(H,1),(H,2)\} \Rightarrow n(E_3) = 2$

$$Pr(E_3) = \frac{n(E_3)}{n(S)} = \frac{2}{12}$$

$E_4 = \{(T,3),(T,4),(T,5),(T,6)\} \Rightarrow n(E_4) = 4$

$$Pr(E_4) = \frac{n(E_4)}{n(S)} = \frac{4}{12}$$

2

- $Pr(E_1^c) = 1 - Pr(E_1) = 1 - \frac{6}{12} = \frac{16}{12}$

- $E_1 \cap E_2 = \{(H,2)\ (H,4), (H,6)\} \Rightarrow n(E_1 \cap E_2) = 3$

$Pr(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{3}{12}$ 4

- $Pr(E_1 \cap E_2^c) = Pr(E_1) - Pr(E_1 \cap E_2)$

$$= \frac{6}{12} - \frac{3}{12} = \frac{3}{12}$$

- $Pr(E_1 \cap E_4^c) = Pr(E_1) - Pr(E_1 \cap E_4)$
$E_1 \cap E_4 = \{(T,4), (T,6)\} \Rightarrow n(E_1 \cap E_4) = 2$
$Pr(E_1 \cap E_4) = \frac{n(E_1 \cap E_4)}{n(S)} = \frac{2}{12}$
$\therefore Pr(E_1 \cap E_4^c) = \frac{6}{12} - \frac{2}{12} = \frac{4}{12}$

- $Pr(E_1 \cap E_2^c) = Pr(E_1) - Pr(E_1 \cap E_2) = \frac{6}{12} - \frac{3}{12} = \frac{3}{12}$

- $Pr(E_1 \cap E_3)^c = 1 - Pr(E_1 \cap E_3)$
$E_1 \cap E_3 = \{(H,2)\} \Rightarrow n(E_1 \cap E_3) = 1 \Rightarrow Pr(E_1 \cap E_3) = \frac{n(E_1 \cap E_3)}{n(S)} = \frac{1}{12}$
$\therefore Pr(E_1 \cap E_3) = 1 - \frac{1}{12} = \frac{11}{12}$

- $Pr(E_1 \cap E_2)^c = 1 - Pr(E_1 \cap E_2) = 1 - \frac{3}{12} = \frac{9}{12}$

- $Pr(E_1^c \cap E_2) = Pr(E_2) - Pr(E_1 \cap E_2) = \frac{6}{12} - \frac{3}{12} = \frac{3}{12}$

- $Pr(E_1 \cup E_2) = Pr(E_1) + Pr(E_2) - Pr(E_1 \cap E_2)$
$$= \frac{6}{12} + \frac{6}{12} - \frac{3}{12} = \frac{9}{12}$$

Ex12: A die is thrown, then for the following events

A: an odd number appears.

B: number appearing is divisible by 3.

Find the probability of $A, B, A^c B$,

Sol: $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S) = 6$

$A = \{1, 3, 5\} \Rightarrow n(A) = 3$

$B = \{3, 6\} \Rightarrow n(B) = 2$

$Pr(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

$A \cap B = \{3\} \Rightarrow n(A \cap B) = \frac{1}{6}$

$Pr(B) = \frac{n(B)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

$Pr(A^c B) = Pr(A^c \cap B) = Pr(B) - Pr(A \cap B)$

$Pr(A^c B) = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

or: $A^c = \{2, 4, 6\} \Rightarrow A^c \cap B = \{6\} \Rightarrow n(A^c \cap B) = 1$

$Pr(A^c B) = Pr(A^c \cap B) = \frac{n(A^c \cap B)}{n(S)}$

Ex13: For Ex16 find $Pr(A^c \cap B^c)$ and $Pr(A \cap B)^c$, H.W

Sol: $Pr(A^c \cap B^c) = 1 - Pr(A \cup B) = 1 - \frac{n(A \cup B)}{n(S)}$

$A \cup B = \{1, 3, 5, 6\} \Rightarrow n(A \cup B) = 4$

$\therefore Pr(A^c \cap B^c) = 1 - \frac{4}{6} = 1 - \frac{2}{3} = \frac{1}{3}$

Ex14: An Urn contains 8 white and 7 red balls, 5 balls are drawn respectively at random,

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E : There will be one red balls
F : There will be at most two red balls.
G : E and F will happen.
H : E but not F.
I : neither E nor F. H.W

Sol: $n=15$, $x=5$, $n_W=8$, $n_R=7$

$$n(S)=C_x^n=\frac{n!}{x!(n-x)!}=C_5^{15}=\frac{15!}{5!\,10!}=\frac{15*14*13*12*11}{5*4*3*2}=3003$$

$$Pr(E)=\frac{n(E)}{n(S)}=\frac{n(1R \text{ and } 4W)}{n(S)}=\frac{n(1R)\cdot n(4W)}{n(S)}=\frac{C_1^7 C_4^8}{C_5^{15}}$$

$$Pr(F)=\frac{n(F)}{n(S)}=\frac{n(1R\,4W \text{ or } 2R\,2W)}{n(S)}$$

$$=\frac{n(1R.4W)+n(2R.2W)}{n(S)}=\frac{n(1R).n(4W)+n(2R).n(2W)}{n(S)}$$

$$=\frac{C_1^7 C_4^8+C_2^7 C_2^8}{C_5^{15}}$$

$$Pr(G)=Pr(E\cap F)=\frac{n(E\cap F)}{n(S)}=\frac{n(1R.4W)}{n(S)}=\frac{n(1R).n(4W)}{n(S)}=\frac{C_1^7 . C_4^8}{C_5^{15}}$$

$$Pr(H)=Pr(EF^c)=Pr(E)-Pr(E\cap F)$$ تعويض تاغ H.W.

EX 15: The resutls in table below are represent a Survey study of technology brands for 210 Persons. If a person is chosen at random.

	Mobile	Tab	
Iphone	65	35	100
Samsung	40	70	110
	105	105	

If a person is chosen at random, what is the probability that this person will has:

[1] Samsung brand. [2] Iphone mobile brand.
[3] tab brand. [4] Iphone tab brand.
[5] Iphone brand. [6] not Iphone or not tab.
[7] Not Iphone tab. [8] not samsung tab.
[9] Not Samsung mobile [10] Not mobile.

Sol: $n(S) = 210$

SaM or SaT

[1] $Pr(Sa) = \frac{n(Sa)}{n(S)} = \frac{110}{210} = \frac{11}{21}$

[2] $Pr(I \text{ and } M) = Pr(I \cap M) = \frac{n(I \cap M)}{n(S)} = \frac{65}{210} = \frac{13}{42}$

TI or TSa

[3] $Pr(T) = \frac{n(T)}{n(S)} = \frac{105}{210} = \frac{1}{2}$

[4] $Pr(I \text{ and tab}) = Pr(I \cap T) = \frac{n(I \cap T)}{n(S)} = \frac{35}{210} = \frac{1}{6}$

[5] $Pr(I) = \frac{n(I)}{n(S)} = \frac{100}{210} = \frac{10}{21}$

[6] $Pr(\text{not } I \text{ or not } T) = Pr(I^c \cup T^c) = 1 - Pr(I \cap T)$

$$= 1 - \frac{n(I \cap T)}{n(S)} = 1 - \frac{1}{6} = \frac{5}{6}$$

$\boxed{7}$ $Pr\{not(I \cap T)\} = Pr(I \cap T)^c = 1 - Pr(I \cap T)$

$$= 1 - \frac{1}{6} = \frac{5}{6}$$

$\boxed{8}$ $Pr(not\ Sa T) = Pr(Sa \cap T)^c = 1 - Pr(Sa \cap T)$

$$= 1 - \frac{n(Sa \cap T)}{n(S)} = 1 - \frac{70}{210} = \frac{140}{210} = \frac{1}{3}$$

$\boxed{9}$ $Pr(not\ Sa M) = Pr(Sa \cap M)^c = 1 - Pr(Sa \cap M)$

$$= 1 - \frac{n(Sa \cap M)}{n(S)} = 1 - \frac{40}{210} = \frac{170}{210} = \frac{17}{21}$$

$\boxed{10}$ $Pr(not\ M) = Pr(M^c) = 1 - Pr(M) = 1 - \frac{n(M)}{n(S)} = 1 - \frac{105}{210} = \frac{1}{2}$

or $Pr(T) = \frac{1}{2}$

Ex5 P 84
Ex6 P 85
Ex7 P 86
Ex8 P 87
Ex9 P 88
Ex11 Ex12 P 89