Chapter Two: The Real Roots of the Equation

2.1 Introduction:

In this chapter we will discuss a number of numerical methods that aim to find an approximate value for a specific root. The equation contains one variable $f(x)=0$. For example, the function can be solved by finding the values of x which makes the function equal to zero and this is done by using numerical methods that need an initial approximate value for the root of the given equation to enable it to target better approximate values for the root and there are many approximate methods for finding the initial roots as well as some methods chosen to find the approximate true value of the root secondly.

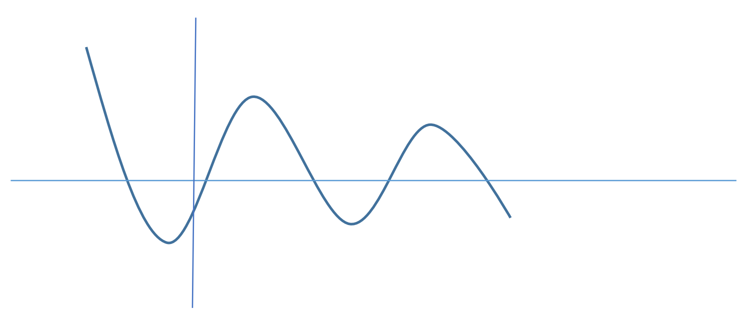
2-2 Location of the roots:

To set the root location there are two ways to specify it.

a) Graphical Method:

This method consists of drawing a graph of the function $y = f(x)$ according to these two cases.

The first case: if the function graph contains only one curve, then the root location is at the intersection of the curve with the x-axis. The intersection may be at one point, which means (there is one root). If there is more than one intersection, it means (there is more than one root).



Example: Find the root of the following equation $f(x) = x + 10$.

If the graph of the function intersects the x-axis at points $(x_1, x_2, \ldots\ldots, x_n)$ Each of these values represents a root of the equation, and therefore these values can be considered as first approximations to the roots.

Example:

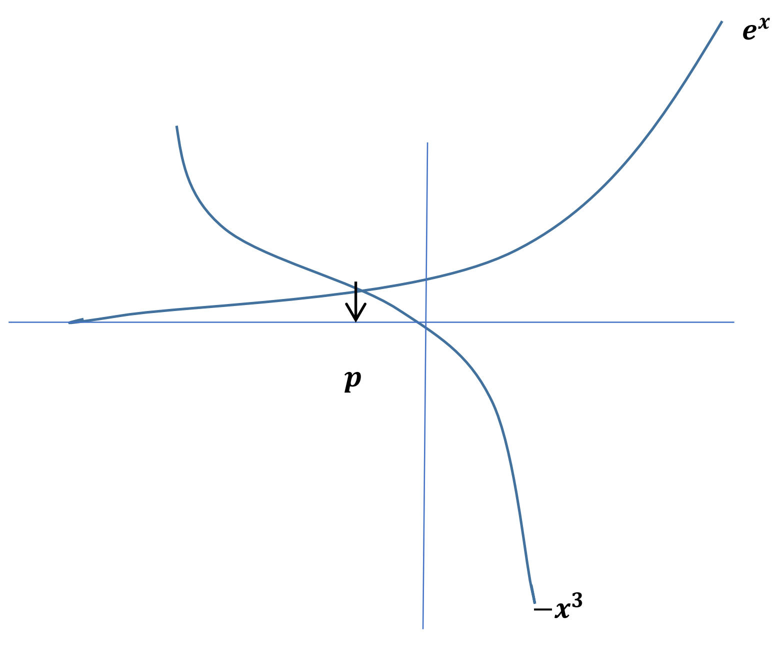
Find the root of the equation $f(x) = e^x + x^3 = 0$

Soluation

We can write the previous equation by dividing it so that:

$$e^x = -x^3$$

Then we draw the two functions as in Figure (2)



b) Second Method : the programmed method

This method depends on the signs of the function values changing at multiple points $\boldsymbol{x_1, x_2 \ldots x_n}$ If two values $f(x_i)$, $f(x_i)$ are different in sign, there is a root between x_i , x_{i+1}.

Example:

Find the root of the equation

$$\boldsymbol{f(x) = x^4 - 7x^3 + 3x^2 + 26x - 10 = 0}$$

In the period (-8, 8)

If we take the division interval h equal to 4, then the sign of the function at the division points is as the follows:

X	-8	-6	-4	-2	0	2	4	6	8
F(x)	+	+	+	+	-	+	-	+	+

There is a roots in the interval (-2, 0), (0, 2), (2, 4), (4, 6)

Next, we study some known numerical algorithms those can be used to find the approximate solutions (roots) for non-linear equations, which are Bisection algorithm, Newton–Raphson algorithm and fixed point algorithm.