Bisection method:

In this method, the interval (a, b) that contains the root and which causes the sign of the function to change on both sides is found. After that, the bisection method is used and the midpoint value is tested in each iteration until the required accuracy is reached.

This method depend on the fact that the function F(x) is a real function, $\boldsymbol{f} \in x_0[\boldsymbol{a}, \boldsymbol{b}]$
Where f is continuous function on the closed interval $[\boldsymbol{a}, \boldsymbol{b}]$ and the condition $f(a)f(b) < 0$ is satisfied There is at least one root in that period$[\boldsymbol{a}, \boldsymbol{b}]$ the point x_0 represent the midpoint of the interval $[\boldsymbol{a}, \boldsymbol{b}]$ as follows:

$$x_0 = \frac{a+b}{2}$$

Then we start testing x_0 according to the conditions

1- If $f(x_0) = 0$, it follows that x_0 is the exact root
2- If $f(x_0) < 0$, $f(b) > 0$ then the exact root belong to (x_0 , b) and we set
a=x_0 , b=b
3- if $f(x_0) > 0, f(a) < 0$ then the exact root belong to (a, x_0) and we set
a=a, b= x_0

We continue iteratively, until we get the following condition is satisfied $|f(x_n) <\in|$ or $|x_{n+1} - x_n| <\in$ or $|b_n - a_n| <\in$

Example: consider that, we have the following equation

$f(x) = x^2 - 2 = 0,$ $x \in [1, 2]$,

Solution

Clearly,$x = \sqrt{2} \approx 1.414$, is the exact root of f on the interval [1,2] $f(2) = 2,$, $f(1) = -1$, this there exists a root in this interval

$a_0 = 1, b_0 = 2$, $x_0 = \frac{a_0+,b_0}{2}$ =1.5, f $(x_0) =$

$0.25 > 0$

So

$a_1 = 1, b_1 = 1.5$, $x_1 = \frac{a_1+,b_1}{2}$ =1.25, f $(x_1) =$

$-0.437 < 0$

So, the iterative errors can be found as follows:

$$E_n = |x_{n+1} - x_n| <\in$$

$$E_1 = |x_1 - x_0| <\in$$

$$E_1 = |1.25 - 1.5| = 0.25 <\in$$

$$x_2 = \frac{a_2+,b_2}{2} = \frac{1.25+,1.5}{2} = 1.375 \quad , f(x_2) = -0.1094 < 0$$

i	a_i	b_i	x_i	$f(x_m)$
0	1	2	1.5	0.25
1	1	1.5	1.25	-0.4375
2	1.25	1.5	1.375	-0.1094

The iterative error,

$$E_2 = |x_2 - x_1| <\in$$

$$E_2 = |1.375 - 1.25| = 0.125 <\in$$

Clearly, it is, still too large, so we have to continue in the iterative processes until we get the convergent condition, $|c_{n+1} - c_n| < \in$, is satisfied .

Or $|b_n - a_n| < \in$

Example: Use the Bisection method to find a solution accurate to $\varepsilon = 0.01$ for the equation $f(x) = x^2 - \frac{1}{2}$.

Solution:

$$x = 0 \Rightarrow f(0) = -\frac{1}{2}$$

$$x = 1 \Rightarrow f(1) = \frac{1}{2}$$

$$x_m = \frac{0+1}{2} = 0.5$$

$$f(0.5) = (0.5)^2 - \frac{1}{2} = -0.25$$

i	X_1	$f(X_1)$	X_2	$f(X_2)$	X_m	$F(X_m)$
1	0	-0.5	1	0.5	0.5	-0.25
2	0.5	-0.25	1	0.5	0.75	0.0625
3	0.5	-0.25	0.75	0.06625	0.625	-0.109379
4	0.625	-0.109379	0.75	0.06625	0.6875	-0.03353125
5	0.6875	-0.0335312	0.75	0.06625	0.71875	0.0166015625
6	0.6875	-0.0335312	0.71875	0.0166015	0.703125	-0.005615