Chapter Three (Roots of Polynomial Equations)

3-1 Introduction

The methods in the previous chapter can be used to calculate the real root of a general function $f(x)$ with numerical coefficients. The polynomial $P_n(x)$ of a variable (x) of degree (n) can be expressed in different ways, including:

3-1-1 General formula:

The general form of a polynomial of degree n for the variable x can be written as:

$P_n(x) = a_0x^n + a_1x^{n-1} + ... + a_{n-1}x + a_n$ Where a_i: Represents the constant (i)

for the values i=0,1,……,n ,n a positive integer representing the degree of a polynomial.

3-1-2 Nested Multiplication:

The Nested Multiplication can be written as:

$$P_n(x) = [\{(a_0x + a_1)x + a_2\}x + ...]x + a_n$$

Example:

Express the following polynomial in Nested Multiplication.

$$P_4(x) = 5x^4 - 2x^3 + 3x^2 - 9$$

Solution

$$P_4(x) = [\{(5\,x - 2)\,x + 3\}\,x + 0]\,x - 9$$

3-1-3 Set of Polynomial Partial formula:

We Can be defined the set of polynomial formula as follows:

$$P_0(x) = a_0$$
$$P_1(x) = xP_0(x) + a_1$$
$$P_2(x) = xP_1(x) + a_2$$
$$\vdots$$
$$P_n(x) = xP_{n-1}(x) + a_n$$

Example:

Express the following polynomial in Set of Polynomial Partial formula.

$$P_4(x) = 8x^4 - 6x^3 + 5x^2 - 2x + 5$$

Then calculate $P_4(2)$.

Solution

$$P_0(x) = 8$$
$$P_1(x) = xP_0(x) - 6$$
$$P_1(x) = 8x - 6$$
$$P_2(x) = xP_1(x) + 5$$
$$P_2(x) = x(8x - 6) + 5$$
$$P_2(x) = 8x^2 - 6x + 5$$
$$P_3(x) = 8x^3 - 6x^2 + 5x - 2$$
$$P_4(x) = 8x^4 - 6x^3 + 5x^2 - 2x + 5$$

When x =2, then

$$P_0(2) = 8$$
$$P_1(2) = 10$$
$$P_2(2) = 25$$
$$P_3(2) = 48$$
$$P_4(2) = 101$$

3-2 Some theories of roots of polynomial

3-2-1 the first theory is the remainder theory:

If a polynomial $P_n(x)$ is divided by a linear quantity $(x-\alpha)$, the resulting polynomial is of degree (n-1) and the remainder is constant r that is:

$$P_n(x) = (x-\alpha)Z_{n-1}(x) + r$$

3-2-2The second theory is the factor theory

If the constant remainder r equals zero when the polynomial $P_n(x)$ is divided by $(x-\alpha)$ the expression, then the expression $(x-\alpha)$ is a factor of the polynomial $P_n(x)$.

3-2-3The third theory is Descartes' theory:

The number of positive roots of a polynomial $P_n(x)=0$ with real coefficients does not exceed the number of changes in the signs of the coefficients $P_n(x)$, and the number of negative roots does not exceed the number of changes in the coefficients $P_n(-x)$.

Exercises:

1- Determine whether x= 4 is a root of the equation.

$$P_3(x) = x^3 - 3x^2 + 8$$

2- Test x = 1, 2, - 4, - 6 to see if it is a root of the previous equation.

3- Find the number of negative and positive roots of the following

equation. $P_5(x) = 4x^5 - 2x^4 - 9x^3 + 6x^2 + 8x - 4$