Chapter Four

Finite Differences

In this chapter we will discuss types of finite differences with their tables, which are considered the basis on which the theory of numerical analysis was established.

The function used here is an equidistant discrete function and is as follows:



There are three types of finite differences:

1- Forward differences.

2- Backward differences.

3- Central differences.

Forward differences:It is symbolized by () the symbol and known as the equation



It is called the first forward difference, and we get the second difference by taking the difference for the first difference.



The differences in rank K can be expressed as follows:



Forward Difference Table

The forward differences of any order can be obtained by subtracting the upper differences from the lower differences in the adjacent column to the left. Note that the location of the odd differences is opposite the middle of the distance between the initial point values, while the even differences are in a straight line opposite the initial values.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| xi | yi |  |  |  |  |
| X0 | y0 |  |  |  |  |
|  |  |  |  |  |  |
| X1 | y1 |  |  |  |  |
|  |  |  |  |  |  |
| X2 | y2 |  |  |  |  |
|  |  |  |  |  |  |
| X3 | y3 |  |  |  |  |
|  |  |  |  |  |  |
| Xn-3 | yn-3 |  |  |  |  |
|  |  |  |  |  |  |
| Xn-2 | yn-2 |  |  |  |  |
|  |  |  |  |  |  |
| Xn-1 | yn-1 |  |  |  |  |
|  |  |  |  |  |  |
| Xn | yn |  |  |  |  |

Example: Write a forward difference table for the following function:



Solution: Substitute the value of x into the equation, extract the values ​​of y, and then find the differences.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| xi | yi |  |  |  |  |
| -1 | -1 |  |  |  |  |
|  |  | 4 |  |  |  |
| 0 | 3 |  | -8 |  |  |
|  |  | -4 |  | 12 |  |
| 1 | -1 |  | 4 |  | 24 |
|  |  | 0 |  | 36 |  |
| 2 | -1 |  | 40 |  |  |
|  |  | 40 |  |  |  |
| 3 | 39 |  |  |  |  |

Homework (1): Make a forward difference table for the following data.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| xi | 3 | 6 | 9 | 12 | 15 |
| yi | 42 | 40 | 39 | 36 | 30 |

Homework (1):Calculate the first forward differences of the following function.

