Theorems of set Operations						
7.	Idempotent Laws	a. $A \cup A = A$		b. $A \cap A = A$		
8.	Associative Laws	a.$(A \cup B) \cup C = A \cup (B \cup C)$		b.$(A \cap B) \cap C = A \cap (B \cap C)$		
9.	Commutative Laws	a.$A \cup B = B \cup A$		b.$A \cap B = B \cap A$		
10.	Distributive Laws	a.$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$		b.$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$		
11.	Identity Laws	a. $A \cup \phi = A$	b. $A \cup U = U$	c. $A \cap \phi = \phi$		d. $A \cap U = A$
12.	Complement Laws	a.$A \cup A^c = U$	b.$A \cap A^c = \phi$	c. $(A^c)^c = A$	d. $U^c = \phi$	e. $\phi^c = U$
13.	De morgans Laws	a.$(A \cup B)^c = A^c \cap B^c$		b.$(A \cap B)^c = A^c \cup B^c$		

Ex.18: Let $U = \{0,1,2,3,4,5,6,7,8,9\}, = \{2,3,4,6,9\}$, $B = \{1,3,5,6,9\}$,
$C = \{0,1,2,4,5,6,7\}$

1. $A \cup A = \{2,3,4,6,9\} = A$, $A \cap A = \{2,3,4,6,9\} = A$

2. $(A \cup B) \cup C = A \cup (B \cup C)$

$(A \cup B) = \{1,2,3,4,5,6,9\} \cup C = \{0,1,2,3,4,5,6,7,9\}$

$A \cup \{0,1,2,3,4,5,6,7,9\} = \{0,1,2,3,4,5,6,7,9\}$

$\therefore (A \cup B) \cup C = A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

$(A \cap B) = \{3,6,9\} \cap C = \{6\}$

$A \cap \{1,5,6\} = \{6\}$

$\therefore (A \cap B) \cap C = A \cap (B \cap C)$

3. $A \cup B = B \cup A$

$\{1,2,3,4,5,6,9\} = \{1,2,3,4,5,6,9\}$

$A \cap B = B \cap A$

$\{3,6,9\} = \{3,6,9\}$

4. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cup \{1,5,6\} = \{1,2,3,4,5,6,9\}$

$\{1,2,3,4,5,6,9\} \cap \{0,1,2,3,4,5,6,7,9\} = \{1,2,3,4,5,6,9\}$

$\therefore A \cup (B \cap C) = (A \cup \mathrm{B}) \cap (A \cup \mathrm{C})$

$A \cap (B \cup C) = (A \cap \mathrm{B}) \cup (A \cap \mathrm{C})$

$A \cap \{0,1,2,3,4,5,6,7,9\} = \{2,3,4,6,9\}$

$\{3,6,9\} \cup \{2,4,6\} = \{2,3,4,6,9\}$

$\therefore A \cap (B \cup C) = (A \cap \mathrm{B}) \cup (A \cap \mathrm{C})$

5. $A \cup \phi = A$

$\{2,3,4,6,9\} \cup \{\ \} = \{2,3,4,6,9\} = A$

$A \cup U = U$

$\{2,3,4,6,9\} \cup \{0,1,2,3,4,5,6,7,8,9\} = U$

$A \cap \phi = \phi$

$\{2,3,4,6,9\} \cap \{\ \} = \phi$

$A \cap U = A$

$\{2,3,4,6,9\} \cap \{0,1,2,3,4,5,6,7,8,9\} = \{2,3,4,6,9\} = A$

6. $A \cup A^c = U$

$A^c = \{0,1,5,7,8\}$

$A \cup A^c = \{2,3,4,6,9\} \cup \{0,1,5,7,8\} = \{0,1,2,3,4,5,6,7,8,9\} = U$

$A \cap A^c = \{2,3,4,6,9\} \cap \{0,1,5,7,8\} = \{\ \}$

$(A^c)^c = \{2,3,4,6,9\} = A$

$(U)^c = \phi \quad , \quad (\phi)^c = U$

7. $(A \cup B)^c = A^c \cap B^c$

$A \cup B = \{1,2,3,4,5,6,9\}$

$(A \cup B)^c = \{0,7,8\}$

$A^c = \{0,1,5,7,8\}$

$B^c = \{0,2,4,7,8\}$

$A^c \cap B^c = \{0,7,8\}$

$\therefore (A \cup B)^c = A^c \cap B^c$

$(A \cap B)^c = A^c \cup B^c$

$A \cap B = \{3,6,9\}$

$(A \cap B)^c = \{0,1,2,4,5,7,8\}$

$A^c \cup B^c = \{0,1,2,4,5,7,8\}$

$\therefore (A \cap B)^c = A^c \cup B^c$

Ordered Pairs and Product Sets

Def. 11: Ordered Pair

An ordered pair consists of two elements, say a and b, in which one of them, say a, is designated as the first element and the other as the second element.

An ordered pair is denoted by (a,b) with $a \neq b \text{ or } a = b$, this means a is not necessarily the same value of b.

Two ordered pairs (a,b) and (c,d) are equal if and only if a=c and b=d.

Ex.19: The ordered pairs (2,3) and (3,2) are different.

Ex.20: The ordered pairs with the same value such as: (2,2), (4,4), (8,8).

Def. 12: Product Sets

Let A and B be two sets. The product set of A and B consists of all ordered pairs (a,b) where $a \in A$ and $b \in B$, It is denote by $A \times B$

i.e. $A \times B = \{(a,b)/a \in A, b \in B\}$

The product set $A \times B$ is also called the "Cartesian product of A and B".

<u>Ex.21:</u> Let $A = \{1,3,5\}$ and $B = \{x,y\}$, Then the product set of A and B is:

$A \times B = \{(1,x),(1,y),(3,x),(3,y),(5,x),(5,y)\}$.

<u>Ex.22:</u> Let $A = \{a,b,c\}$ Then
$A \times A = \{(a,a),(a,b),(a,c),(b,b),(b,a),(b,c),(c,c),(c,a),(c,b)\}$

<u>Properties of Cartesian Product:</u>

1. $A \times B \neq B \times A$
2. If A of (n) elements, and B of (m) elements, then $A \times B$ has (nm) elements.
3. If either A or B is the null set, then $A \times B$ is also the null set.
4. If either A or B is infinite and the other is not empty, then $A \times B$ is infinite.
5. $A \times B = B \times A$ if A=B.

<u>Def. 13</u>: Relations

A relation R from A to B is a subset of $A \times B$

i.e. $R \subset A \times B$.

<u>Ex.23:</u> Let $A = \{1,3,5\}$ and $B = \{a,b\}$

$A \times B = \{(1,a),(1,b),(3,a),(3,b),(5,a),(5,b)\}$.

$R = \{(1,a),(1,b),(3,a),(5,b)\}$

Then R is a relation from A to B.

<u>Def. 14</u>: Inverse Relation

Every relation R from A to B has an inverse relation R^{-1} from B to A which is defined by:

$R^{-1} = \{(b, a) / (a, b) \in R\}$

<u>Ex.24:</u> Let $A = \{1,2,3\}$ and $B = \{c, d, e\}$

$A \times B = \{(1, c), (1, d), (1, e), (2, c), (2, d), (2, e), (3, c), (3, d), (3, e)\}$.

$R = \{(1, c), (1, d), (1, e), (2, e)\}$

Then the inverse relation of R is:

$R^{-1} = \{(c, 1), (d, 1), (e, 1), (e, 2)\}$

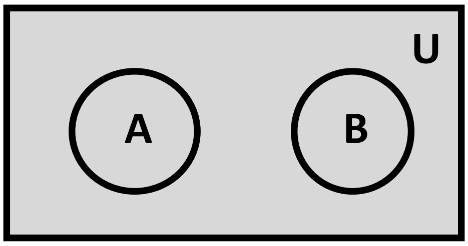
<u>Def. 15</u>: Venn Diagrams

It is, at times, convenient to represent sets by diagrams.In these diagrams, known as:

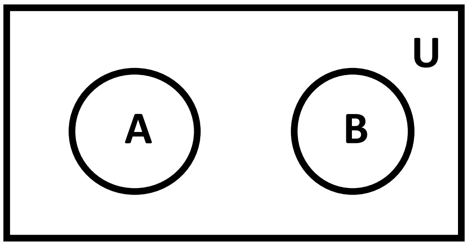
"Venn Diagrams". The universal set U is represented by a rectangle, and subsets are represented by circles inside the rectangle.

In this diagram, A&B are subsets of the universal set U.

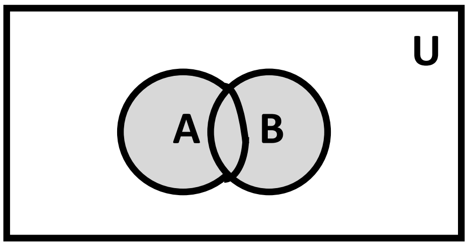
<u>Ex.25</u>: Universal Set U



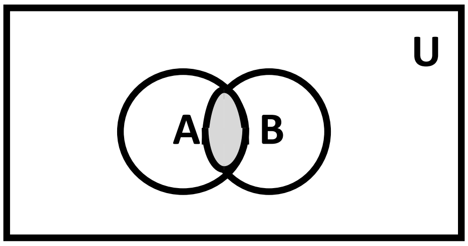
<u>Ex.26</u>: A, B mutually exclusive set



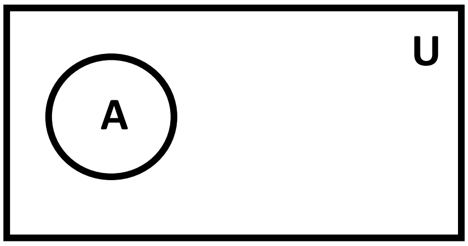
<u>Ex.27</u>: $A \cup B$



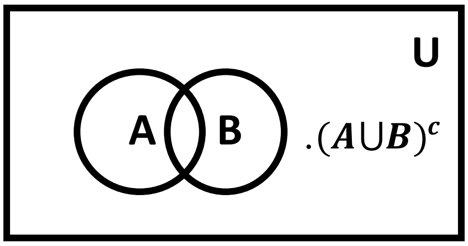
<u>Ex.28</u>: $A \cap B$



<u>Ex.29</u>: A^c



<u>Ex.30</u>: $.(A \cup B)^c$



<u>Ex.31</u>: $A/B = A - B$