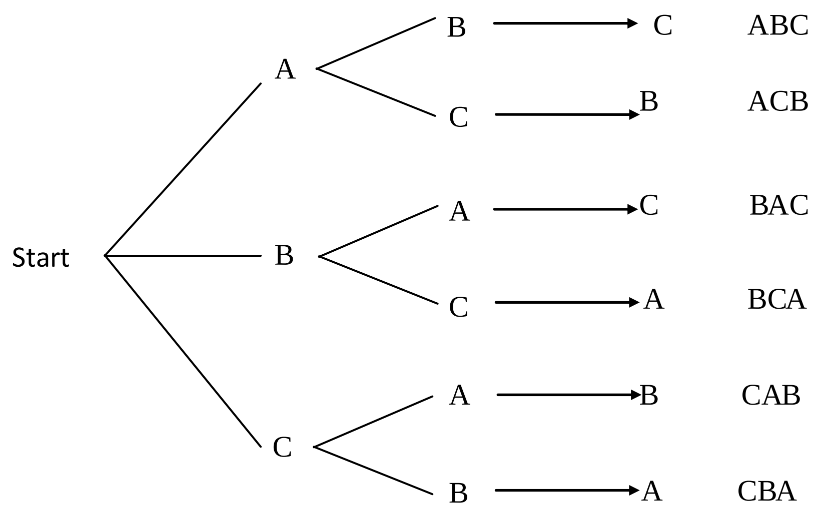
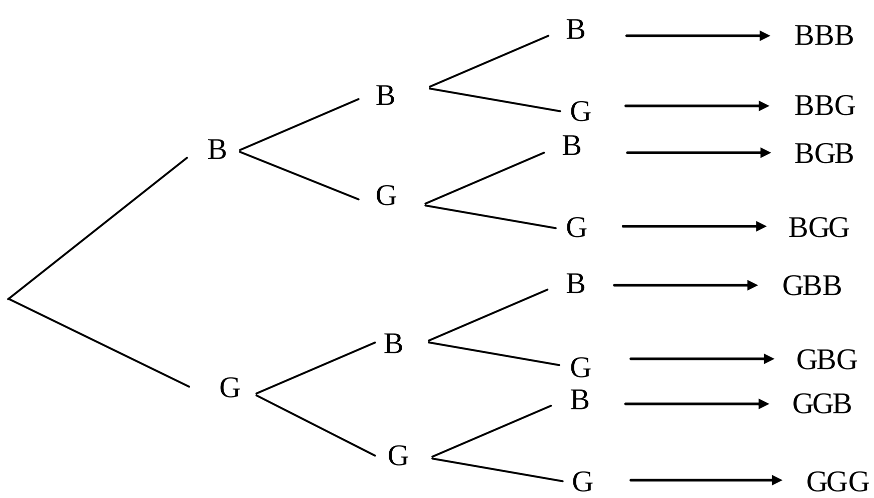
<u>Def. 16</u>: Tree Diagrams

A tree diagram provide an organized way all the possible outcomes of a sequence of experiments where each experiment can occur in a finite number of ways. Branches represent possible outcomes. The tree diagram method is useful when the sample space is established through a series of steps or stages.

<u>Ex.32</u>: In how many ways can (3) books denoted by A, B and C be arranged in order on a shelf?



<u>Ex.33</u>: Suppose that a couple is planning to have three children. In how many ways can this happen?



<u>Def. 17</u>: Arrangements

If (n) objects are taken out of the total number of (m) objects to form groups with attention being paid to the order of objects in each group, then these groups are called "arrangements". For instance, by taking two letters out of the four letters a , b , c , d we can form the following (12) arrangements.

4	3

$4 \times 3 = 12\, ways$

ab	ba	ca	da
ac	bc	cb	db
ad	bd	cd	dc

<u>Ex.34</u>: In how many ways can the letters a , b and c be arranged in a row.

3	2	1

$3 \times 2 \times 1 = 6\, ways$

Abc	acb
Bac	bca
Cab	cba

<u>Ex.35</u>: How many 2 digit numbers can be formed from the seven digit numbers 1,2,3,4,5,6,7.

7	6

$7 \times 6 = 42\, ways$

<u>Ex.36</u>: The seven persons can arrange themselves in a row is:

7	6	5	4	3	2	1

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7! = 5040\, ways$

<u>Ex.37</u>: In how many ways can (3) boys and (2) girls sit in a row?

The five persons can sit in a row

5	4	3	2	1

$5 \times 4 \times 3 \times 2 \times 1 = 5! = 120\ ways$

<u>Def. 18</u>: Permutation

A permutation of a number of objects is any arrangement of these objects in a definite order.

<u>Theorem 14</u> (Permutation of n things, all together)

The number of permutations of a set of (n) different objects, taken all together, is $n!$.

<u>Def. 19</u>:

An arrangement of (r) objects, taken from a set of (n) objects is called a permutation of the (n) objects taken (r) at a time. The total number of such permutation is denoted by: $P_r^n\ ,\ 0 \le r \le n$.

<u>Theorem 15</u> (Permutation of n things, r at a time) without repetitions

The number of permutations of a set of (n) different objects, taken r at a time, **without repetitions**, is:

$$P_r^n = \frac{n!}{(n-r)!} \quad , \qquad 0 \le r \le n$$

<u>Theorem 16</u> (Permutation when repetition is allowed)

When the number of object is (n) and we have (r) to be selection of object, then choosing an object can be in (n) different ways. Thus, the permutation of objects when repetition is allowed will be equal to:

$$n * n * n \dots .(r\ times) = n^r$$

<u>Ex.38</u>: How many (2) digit numbers can be formed from the seven digit numbers? If repetitions is not allowed.

- **1st method**

7	6

$7 \times 6 = 42\, ways$

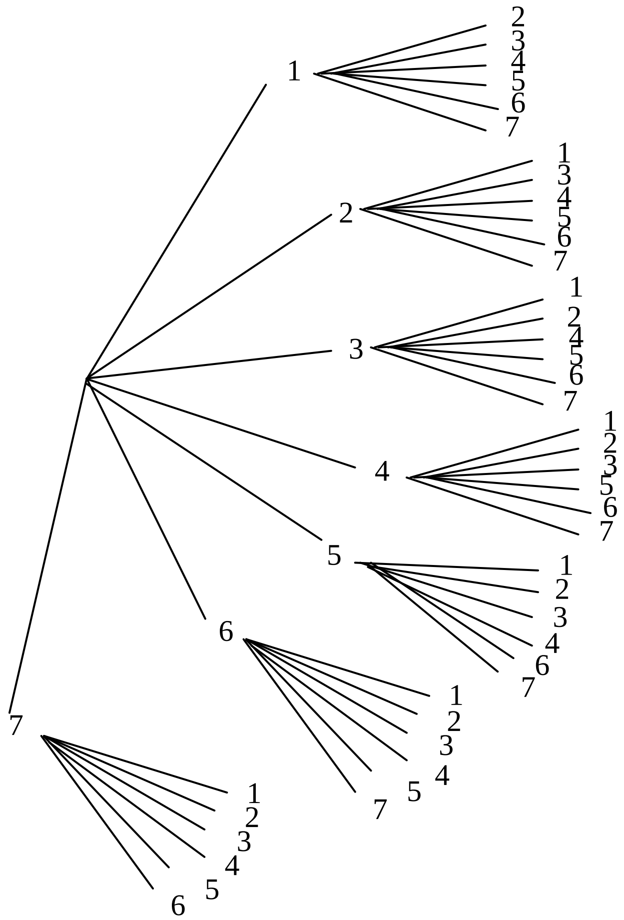
- **2nd method**

$n = 7 \ \ , \ \ r = 2$

$$P_r^n = \frac{n!}{(n-r)!}$$

$$P_2^7 = \frac{7!}{(7-2)!} = \frac{7!}{5!} = \frac{7 \times 6 \times 5!}{5!} = 42$$

- 3rd method



Ex.39: Solve **Ex.38** if repetition is allowed?

- **1st method**

7	7

$7 \times 7 = 49\, ways$

- **2nd method**

$n * n = 7 * 7 = 49 \quad or \quad n^r = 7^2 = 49$

- 3rd method

Ex.40: How many (4) digit numbers can be formed from the digit numbers 1,2,3,4? if repetitions is not allowed.

- **1st method**

4	3	2	1

$4 \times 3 \times 2 \times 1 = 24\, ways$

- **2nd method**

$n! = 4! = 4 \times 3 \times 2 \times 1 = 24$

- **3nd method**

$n = 4 \quad , \quad r = 4$

$$P_r^n = \frac{n!}{(n-r)!}$$

$$P_4^4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4!}{1} = 4! = 4 \times 3 \times 2 \times 1 = 24$$

- 4th method

Theorem 17 (Permutation of multi-sets)

Given a set of (n) objects having (n_1) elements alike of one kind, and (n_2) elements alike of another kind, and (n_3) elements alike of a third kind, and so on for (k) kinds of objects, then the number of permutations of the (n) objects, taken all together is:

$$\frac{n!}{n_1!\ n_2! \ldots \ldots n_k!}$$

Where $n_1 + \ n_2 + \cdots + n_k = n$

Ex.41: How many arrangements can be made of the letters of the word (Mississippi), taken all together?

Since there are 1m , 4i , 4s , 2p and the total of letters is 11, then

$$\frac{n!}{n_1!\ n_2! \ldots \ldots n_k!} = \frac{11!}{1!\ .4!\ .4!\ .2!} = 34650$$

Remark: The number of arrangements of (n) different objects in a circle is given by:

$$(n-1)!\ , n \geq 1$$

Ex.42: What is the number of ways in which (6) persons can be seated in a circle?

$(n-1)! = (6-1)! = 5! = 120$

Remark: The number of arrangements of (n) different objects in a row is given by: $n!$

Ex.43: What is the number of ways in which (6) books can be arranged side by side?

$n! = 6! = 720$

6	5	4	3	2	1

$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720\, ways$

Ex.44: What is the number of ways in which (6) persons can be seated in a row if a certain (2) of them must sit side by side?

$n! = 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$n! = 2! = 2 \times 1 = 2$

$120 \times 2 = 240\, ways$