<u>Def. 20</u>: Combinations

A combination is a selection of objects considered without regard to their order.

<u>Theorem 18</u> (Combinations of n things, r at a time)

The number of combinations of a set of (n) different objects, taken r at a time, is:

$$C_r^n = \frac{n!}{r!\ (n-r)!} \quad , \qquad 0 \le r \le n$$

Corollary: $C_{n-r}^n = C_r^n$

<u>Ex.45</u>: How many committees of (3) can be taken from (8) people?

$n = 8 \ \ , \ r = 3$

$$C_r^n = \frac{n!}{r!\,(n-r)!}$$

$$C_3^8 = \frac{8!}{3!\,(8-3)!} = \frac{8!}{3!\,5!} = \frac{8 \times 7 \times 6 \times 5!}{3 \times 2 \times 1 \times 5!} = 56$$

<u>Ex.46</u>: How many committees of (3) can be chosen from the {Ali, Mohammed, Faris, Saif, Taha)

$n = 5 \ \ , \ r = 3$

$$C_r^n = \frac{n!}{r!\,(n-r)!}$$

$$C_3^5 = \frac{5!}{3!\,(5-3)!} = \frac{5!}{3!\,2!} = \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} = 10$$

Theorem 19 **Pascal's Rule**

$$C_r^{n+1} = C_{r-1}^{n} + C_r^{n} \quad , \qquad 1 \leq r \leq n$$

Ex.47: prove theorem (18) if $n = 6 \ \ , \ r = 3$?

The right side

$$C_r^{n+1} = C_3^7 = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times 4!}{3! \times 4!} = 35$$

The left side

$$C_{r-1}^{n} = C_2^6 = \frac{6!}{2!4!} = \frac{6 \times 5 \times 4!}{2! \times 4!} = 15$$

$$C_r^{n} = C_3^6 = \frac{6!}{3!3!} = \frac{6 \times 5 \times 4 \times 3!}{3! \times 3!} = 20$$

$$C_{r-1}^{n} = C_2^6 + C_r^n = C_3^6$$

$$15 + 20 = 35$$

This implies that: $C_r^{n+1} = C_{r-1}^{n} + C_r^{n}$

Problems

1. Which of the following sets are finite:

a) The months of the year.

b) {1,2,3,…,99,100}.

c) The number of people living on earth.

d) The set R of real numbers.

2. Are the following sets equal? $E = \{x \,/\, x^2 - 3x = -2\}, F = \{2,1\}, G = \{1,2,2,1\}$.

3. Let $U = \{0,1,2,\dots,9\}, A = \{1,2,5\}, B = \{2,3,4\}, C = \{3,5,7\}, D = \{1,2,4,6\}, E = \{0,2,4,6,8\}, F = \{1,3,5,7,9\}$.

a) Find $A \cup B, A \cap B, A \cap D, C \cap D, E \cap F, B \cup C, E \cup F, (A \cup B) \cap (C \cup D)$, $A^c \cap B^C, A^c \cup B^c, (E \cup F) \cap (C \cup D), A - B, B^c \cap A$.

b) Satisfy Associative Laws for A,B,C?

4. Let $A = \{a, b, c\}$. Write the product set $A \times A$?

5. Let $A = \{a, b\}, B = \{2,3\}, C = \{3,4\}$. Find:

a) $A \times (B \cup C)$

b) $(A \times B) \cup (A \times C)$

c) $A \times (B \cap C)$

d) $(A \times B) \cap (B \times C)$

6. Shade the following sets in the Venn diagram:

a) B^c b) $(A \cup B)^c$ c) $(B/A)^c$ d) $A^c \cap B^c$

7. How many license plates can be made using (2) letters followed by a (3) digit number?

8. Given the digits 1,2,3,4 and 5, find how many 4-digite numbers can be formed from them:

a) If no digit may be repeated.

b) If repetitions of a digit are allowed.

c) If the number must be odd, without any repeated digit.

9. In how many ways can (7) persons arranged themselves:

a) In a row of chairs.

b) A round a circular table.

10. A student is to answer (8) out of (10) questions on an exam.

a) How many choices has he?

b) How many if he must answer the first (3) questions?

c) How many if he must answer at least (4) of the first (5) questions?

11. Let $S = \{a, b, c\}, T = \{b, c, d\}, W = \{a, d\}$. Construct the tree diagram of $S \times T \times W$ and then find $S \times T \times W$.