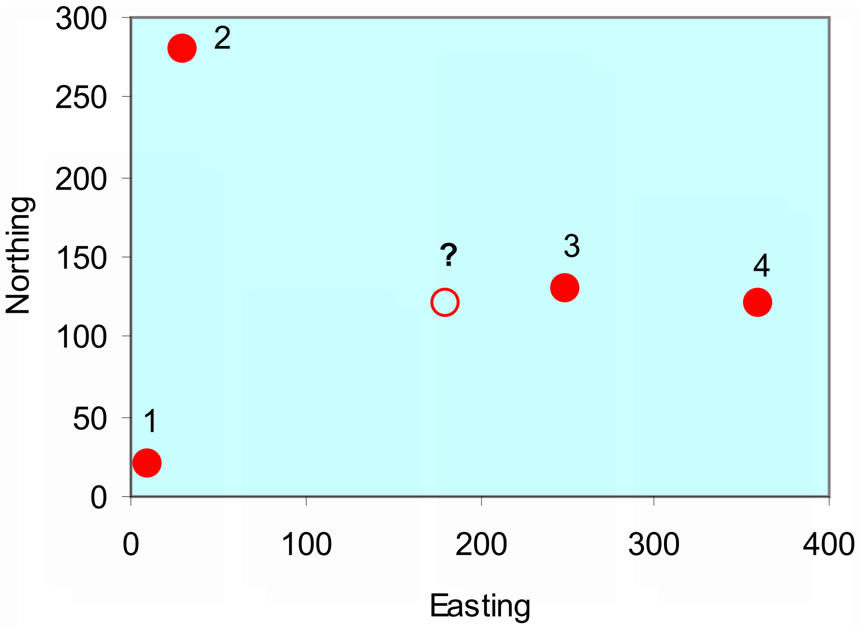
STATIONARY MATRIX FORMULATION

$$\begin{bmatrix} \mathrm{Cov}(\mathbf{s}_1,\mathbf{s}_1) & \cdots & \mathrm{Cov}(\mathbf{s}_k,\mathbf{s}_1) & 1 \\ \vdots & \cdots & \vdots & 1 \\ \mathrm{Cov}(\mathbf{s}_1,\mathbf{s}_k) & \cdots & \mathrm{Cov}(\mathbf{s}_k,\mathbf{s}_k) & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_k \\ \mu \end{bmatrix} = \begin{bmatrix} \mathrm{Cov}(\mathbf{s}_0,\mathbf{s}_1) \\ \vdots \\ \mathrm{Cov}(\mathbf{s}_0,\mathbf{s}_K) \\ 1 \end{bmatrix}$$

$$Z^*_{OK}(\mathbf{s}_0) = \begin{bmatrix} Z(\mathbf{s}_1) \\ \vdots \\ Z(\mathbf{s}_k) \\ 0 \end{bmatrix}^T \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_k \\ \mu \end{bmatrix}$$

$$\sigma^2_{OK}(\mathbf{s}_0) = \mathrm{Cov}(\mathbf{s}_0,\mathbf{s}_0) - \begin{bmatrix} \mathrm{Cov}(\mathbf{s}_0,\mathbf{s}_1) \\ \vdots \\ \mathrm{Cov}(\mathbf{s}_0,\mathbf{s}_k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_k \\ \mu \end{bmatrix}$$

EXERCISE



SAMPLE

Index	Easting	Northing	Attribute
1	10	20	40
2	30	280	130
3	250	130	90
4	360	120	160
?	180	120	

$$\mathrm{Cov}(\mathbf{h}) = 2000e^{-\mathbf{h}/250}$$

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EXERCISE

Apply ordinary kriging to the previous sample to do the following:

A. Find the estimation weights.
B. Note any interesting peculiarities in the weight values.
C. Use the data and weights to evaluate the estimate.
D. Calculate the estimation variance.

Compare to the results obtained using simple kriging.

A. ORDINARY KRIGING WEIGHTS

Distance matrices:

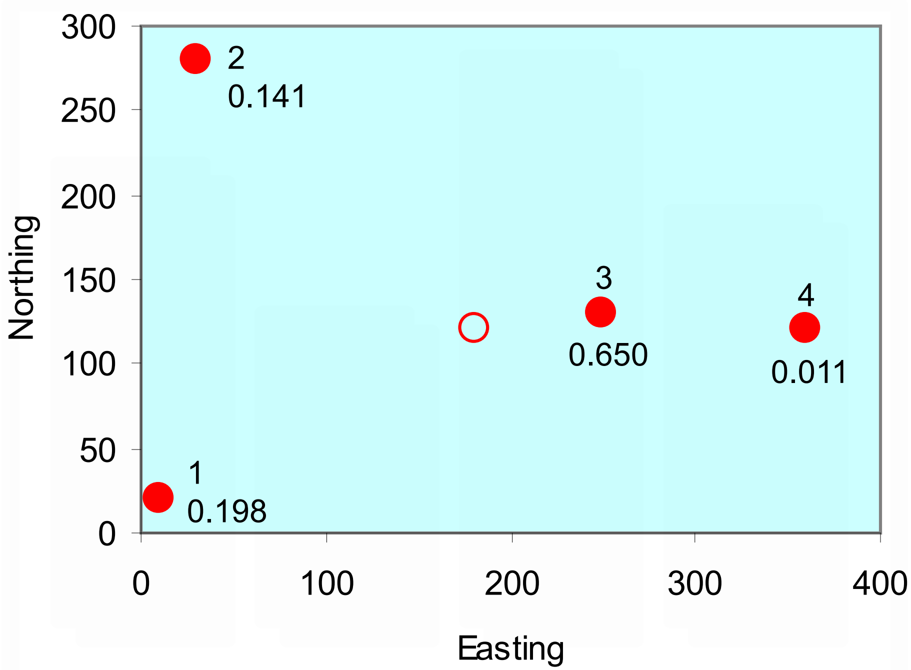
$$\begin{bmatrix} 0 & & & \\ 260.8 & 0 & & \\ 264.0 & 266.3 & 0 & \\ 364.0 & 366.7 & 110.4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 197.2 & 219.3 & 70.7 & 180.0 \end{bmatrix}^T$$

Weights and Lagrange multiplier:

$$\begin{bmatrix} 2000 & & & & \\ 704.8 & 2000 & & & \\ 695.6 & 689.4 & 2000 & & \\ 466.4 & 461.2 & 1285.8 & 2000 & \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 908.7 \\ 831.8 \\ 1507.2 \\ 973.6 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.198 \\ 0.141 \\ 0.650 \\ 0.011 \\ -42.714 \end{bmatrix}$$

B. WEIGHT COMPARISON



Weight	SK	OK
1	0.185	0.198
2	0.128	0.141
3	0.646	0.650
4	-0.001	0.011
Sum	0.958	1.000

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C. ESTIMATES

Using the subscripts *OK* to denote ordinary kriging and *SK* for simple kriging,

$$z^*_{OK}(\mathbf{s}_0) = \begin{bmatrix} Z(\mathbf{s}_1) \\ \vdots \\ Z(\mathbf{s}_k) \\ 0 \end{bmatrix}^T \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_k \\ -\mu \end{bmatrix}$$

$$z^*_{OK}(180{,}120) = \begin{bmatrix} 40 \\ 130 \\ 90 \\ 160 \end{bmatrix}^T \begin{bmatrix} 0.198 \\ 0.141 \\ 0.650 \\ 0.011 \end{bmatrix}$$

$$z^*_{OK}(180{,}120) = 86.6$$

$$z^*_{SK}(180{,}120) \text{ was } 86.7$$

D. ESTIMATION VARIANCE

$$\sigma^2_{OK}(\mathbf{s}_0) = \mathrm{Cov}(\mathbf{s}_0, \mathbf{s}_0) - \begin{bmatrix} \mathrm{Cov}(\mathbf{s}_0, \mathbf{s}_1) \\ \vdots \\ \mathrm{Cov}(\mathbf{s}_0, \mathbf{s}_k) \\ 1 \end{bmatrix}^T \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_k \\ \mu \end{bmatrix}$$

$$\sigma^2_{OK}(180{,}120) = 2000 - \begin{bmatrix} 908.6 \\ 831.8 \\ 1507.2 \\ 973.6 \\ 1 \end{bmatrix}^T \begin{bmatrix} 0.198 \\ 0.141 \\ 0.650 \\ 0.011 \\ -42.714 \end{bmatrix}$$

$\sigma^2_{OK}(180{,}120) = 754.7$ $\qquad$ $\sigma^2_{SK}(180{,}120)$ was 752.9

μ (–42.714) provides the variation in σ^2 per unit of change in the constraint. The change in the sum of weights from OK to SK is 0.042. Hence the reduction of −1.8 (752.9 − 754.7) agrees with the fluctuation in σ^2 (0.042·(−42.724)).

PROPERTIES

Ordinary kriging shares the same properties with simple kriging. In addition,

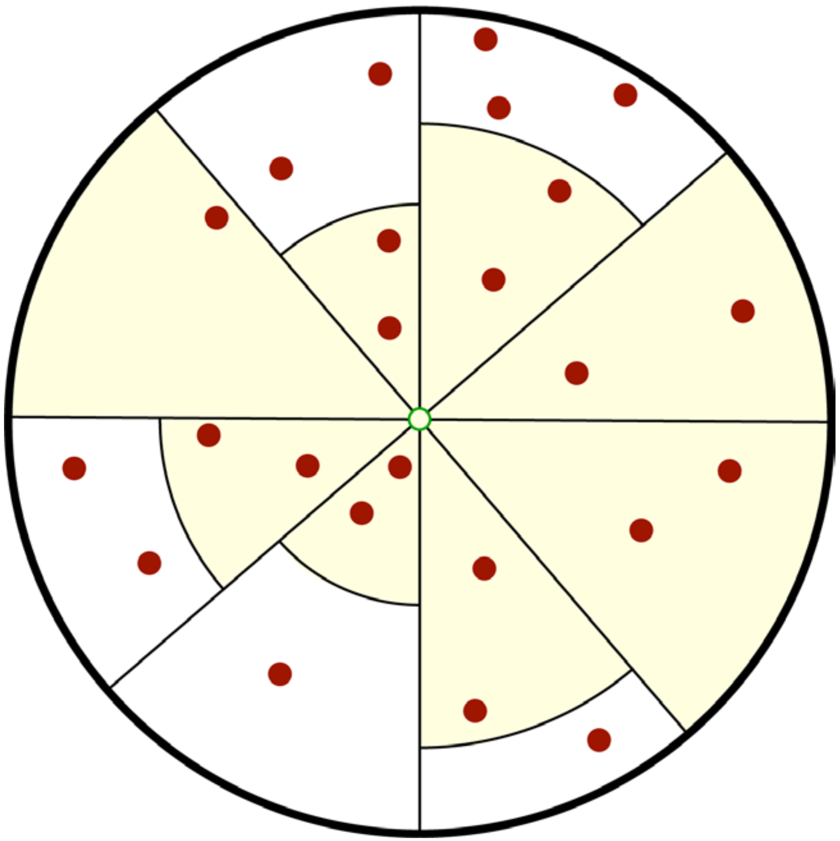
- $\sigma_{OK}^2(\mathbf{s}_0) \geq \sigma_{SK}^2(\mathbf{s}_0)$
- For the trivial case of considering only one observation in the calculation of the estimate,

$$\sigma_{OK}^2(\mathbf{s}_0) = 2 \cdot \gamma(\mathbf{h}_{01}),$$

  where $\mathbf{h}_{01}$ is the distance from the observation, $z(\mathbf{s}_1)$, to the estimation location, $\mathbf{s}_0$.

SEARCH NEIGHBORHOOD

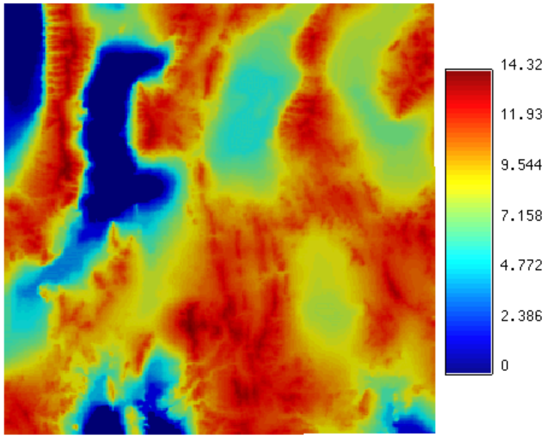
- Because of the screen effect and numerical instabilities, it is recommended that only the closest observations to the estimation location be used.
- Three observations are a reasonable bare minimum and 25 are more than adequate.
- Use octant search to further ensure good radial distribution.



SUBSAMPLE SIZE SENSITIVITY FOR THE CASE OF THE UNCF DATASET

k	λ_1	λ_2	λ_3	λ_4	λ_5	$\sum_{i=6}^{k} \lambda_i$	z^*_{OK}	σ^*_{OK}
5	0.351	0.311	0.043	0.157	0.137	---	0.766	0.159
7	0.358	0.328	0.047	0.157	0.137	-0.047	0.750	0.159
8	0.356	0.325	0.050	0.178	0.165	-0.074	0.767	0.158
10	0.355	0.324	0.050	0.178	0.170	-0.077	0.756	0.158
15	0.355	0.324	0.051	0.178	0.169	-0.077	0.753	0.158
20	0.355	0.324	0.052	0.178	0.169	-0.078	0.754	0.158
25	0.355	0.324	0.052	0.178	0.169	-0.078	0.758	0.158

THE ELY WEST SAMPLE



This is a semi-artificial, exhaustive dataset that will be used extensively to illustrate and compare several methods.

The data are transformed digital elevations in an area in Nevada with extreme cases of continuity: flat dry lakes surrounded by rugged sierras running roughly north-south.

The sampling is perfectly regular on a 286 by 286 grid, making 81,796 observations with a Gaussian anisotropic semivariogram (p. 124): $\gamma_E(\mathbf{h}) = 0.1 + 11 \cdot \text{Gaussian}(\text{N10E}, 80, 34)$.

The exhaustive sample will be used solely for comparisons. All calculations will be done employing the same subset of size 40. Based partly on crossvalidation, its semivariogram is

$$\gamma(\mathbf{h}) = 0.1 + 13 \cdot \text{Gaussian}(\text{N15E}, 80, 35).$$

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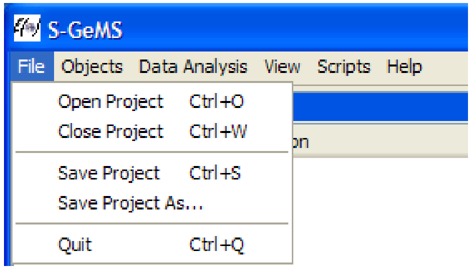
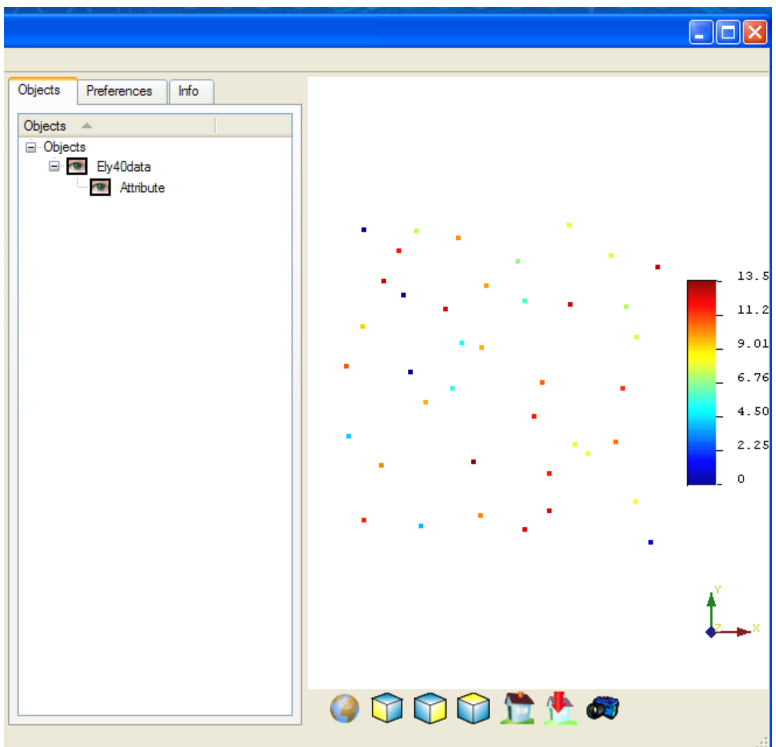
ORDINARY KRIGING WITH SGeMS
A. LOAD SAMPLE

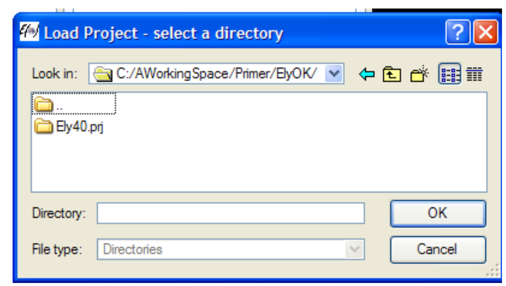


Load data according to instructions in chapter 4.

If data have been previously loaded and saved:

- click on <File>;
- click on <Open Project>;
- select folder and click <OK>.



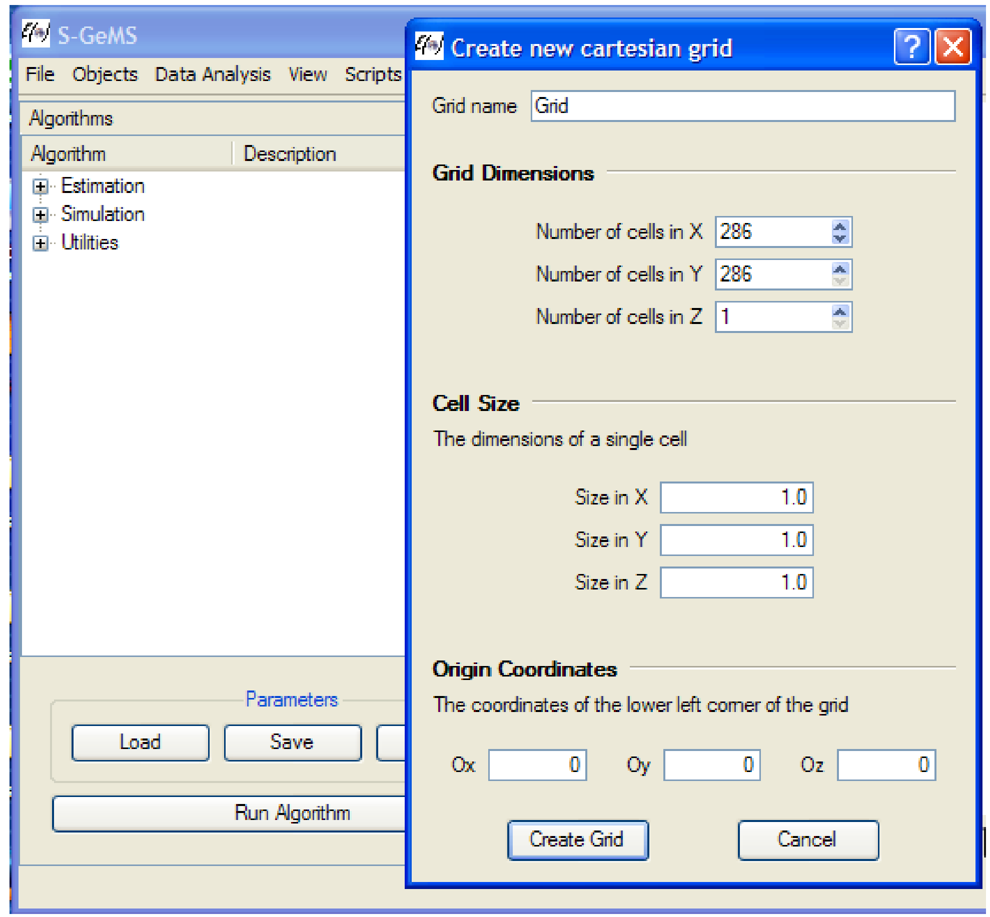


Loading is complete.

To post data, operate on <Object> and <Preferences>.

B. CREATE GRID FOR STORING RESULTS

- Click on <Objects>;
- click on <New Cartesian Grid>;
- fill in options;
- click on <Create Grid>.



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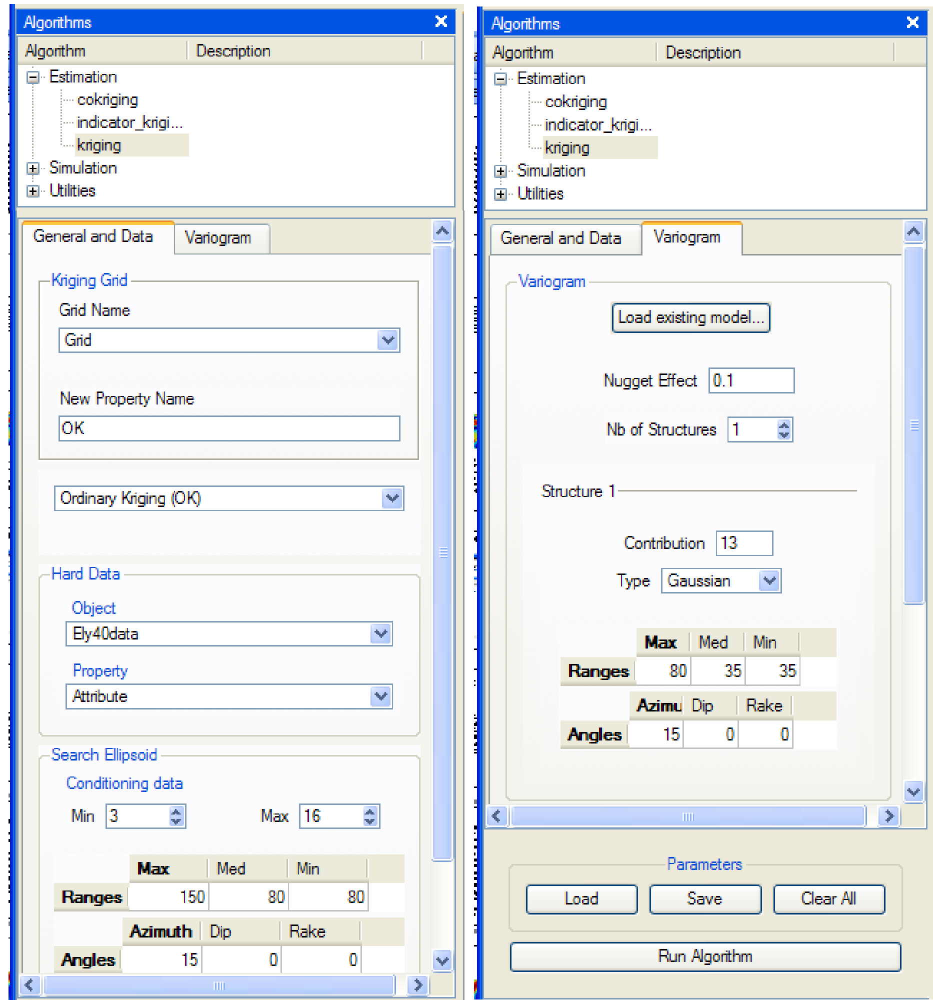
C. SET PARAMETERS

- Click on <Estimation> and then on <kriging>.
- Complete the form. To be able to do that, it is required to previously have modeled the semivariogram.
- Click on <Run Algorithm> when ready.



You may want to click on <Save> to store all parameters as *.par file for further reference.

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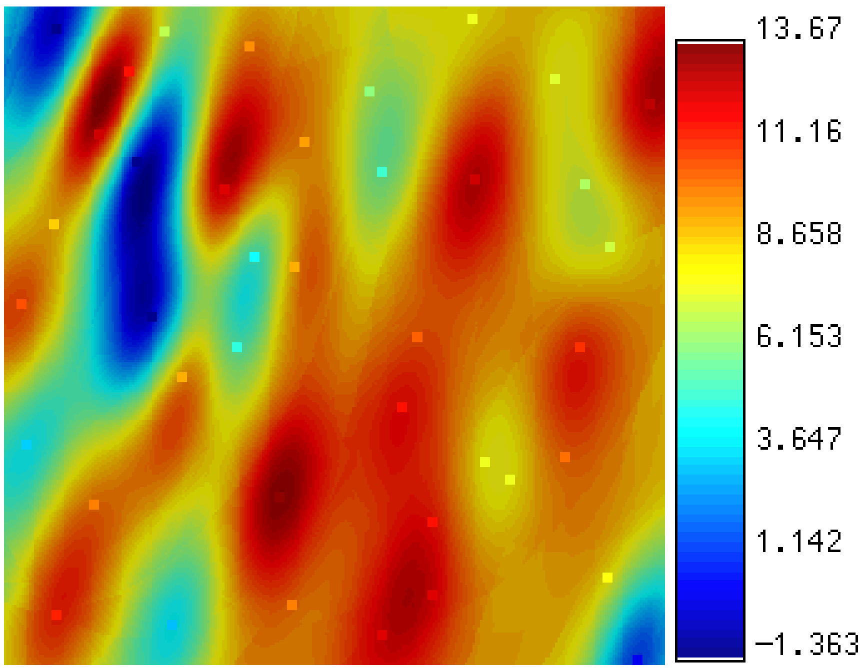
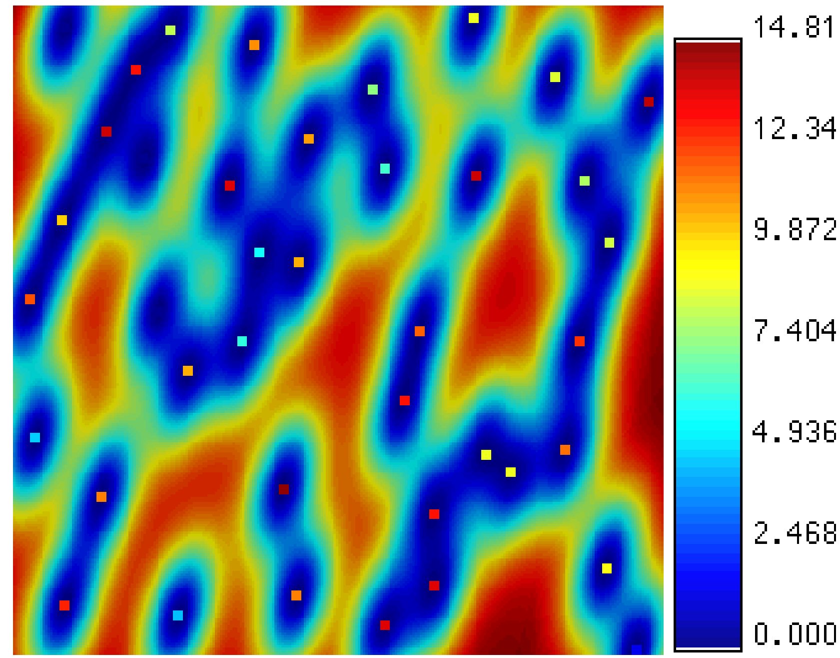
D. RESULTS



z^*_{OK}



σ^2_{OK}

Click on camera icon ( ) at lower right corner of screen to save maps as electronic files.

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10. UNIVERSAL KRIGING

INTRODUCTORY REMARKS

There are several instances of attributes—water depth near the shore, temperature in the upper part of the Earth's crust—that have a clear, systematic variation. Hence, models that presume constancy of the mean are inadequate for the characterization of such attributes.

Universal kriging is a full generalization of simple kriging that takes to the limit the ordinary kriging improvement. Universal kriging removes both the requirement that the attribute must have a constant mean and that the user must know this constant mean.

As the model complexity goes up, unfortunately so does uncertainty. Thus, universal kriging should not be used indiscriminately.

Universal kriging is also known as kriging with trend.