2-2-3 Secant Method (False position method)

It is an iterative method that depends on initial values that are initially assumed and then we calculate the root in each iteration

Definition: - The point x_0 which belongs to the domain of the function g is called a **fixed point** for g, iff $g(x_0) = x_0$ It is calculated according to the formula

$$x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)} \quad i = 0\ , 1, 2, \dots . n$$

We continue in the iterative process until we get:

$$E_1 = |x_1 - x_0| < \epsilon$$

Example 9: - Find the approximate root of the following equation

$$f(x) = 3x^2 - x\text{-}7$$

by secant method with $\epsilon = 0.01$ *where* $x_0 = 1.2\ , x_{-1} = 1$

for three iterative steps (find only $x_1, x_2\ , x_3$)

Solution:-

$$f(x_0) = -3.88 \quad , f(x_{-1}) = -5$$

$$x_1 = x_0 - \frac{f(x_0)(x_{-1} - x_0)}{f(x_{-1}) - f(x_0)}$$

$$x_1 = 1.2 - \frac{-3.88(1 - 1.2)}{-5 - (-3.88)} = 1.8928571$$

$$E_1 = |x_1 - x_0| < \epsilon$$

$$= |1.6928571 - 1.2| = 0.6928571 > \epsilon$$

We continue in the iterative process

$$x_2 = x_1 - \frac{f(x_1)(x_0 - x_1)}{f(x_0) - f(x_1)}$$

$$f(x_1) = f(1.892857) = 1.85586735\ , f(x_0) = -3.88$$

$$x_2 = 1.892857 - \frac{1.85586735\ (-0.692857)}{-3.88 - (1.85586735)} = 1.668680$$

$$E_2 = |x_2 - x_1| < \in$$

$$= |1.668680 - 1.8928571| = 0.224177 > \in$$

We continue in the iterative process

$$x_3 = x_2 - \frac{f(x_2)(x_1 - x_2)}{f(x_1) - f(x_2)}$$

$$f(x_2) = f(1.668680) = -0.31520211\ , f(x_1) = f(1.892857) = 1.85586735$$

$$x_3 = 1.668680 - \frac{-0.31520211\ (1.892857 - 1.668680)}{1.85586735 - (-0.31520211)} = 1.701227$$

$$E_3 = |x_3 - x_2| < \in$$

$$= |1.701227 - 1.668680| = 0.032547 > \in$$

H.W

1-Find the approximate root of the following equation

$$f(x) = x - e^{-x}$$

by secant method with $\in = 0.0001$, [2,3]

2- Find the approximate root of the following equation

$$f(x) = xe^{-x} - 1$$

by secant method with $\in = 0.05$, [0,1]