The Definition of the derivative:

The derivative of the function f is the function f' whose value at x is defined by the eq.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$ whenever the limit exists

Example 1: Use the definition of the derivative to find the derivative of the function

$$f(x) = 3x - 4$$

Sol: $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{3(x+\Delta x)-4-(3x-4)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{3x+3\Delta x-4-3x+4)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{3\Delta x}{\Delta x} = \lim_{\Delta x \to 0} 3 = 3$$

Example 2: Use the definition of the derivative to find the derivative of the function

$$f(x) = x^2$$

Sol: $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2-2x\Delta x+(\Delta x)^2-x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(2x+\Delta x)}{\Delta x} = 2x$$

H.W: Use the definition of the derivative to find the derivative of the following functions:

1- $f(x) = 2x - 1$ 2- $f(x) = (x+1)^2$ 3- $f(x) = x^2 - 1$

4- $f(x) = \sqrt{x}$ 5- $f(x) = \frac{x}{x-9}$ 6- $f(x) = x^3$

Derivative Formulas

Powers of x rule:

If $f(x) = x^n$, then $f'(x) = n(x)^{n-1}$

Constant rule:

If $f(x) = C$, then $f'(x) = 0$

Coefficient rules:

If $f(x) = c \cdot u(x)$, then $f'(x) = c \cdot u'(x)$

If $f(x) = k \cdot x^n$, then $f'(x) = kn(x)^{n-1}$

If $f(x) = kx$, then $f'(x) = k$

Sum rule:

If $f(x) = u(x) + v(x)$, then $f'(x) = u'(x) + v'(x)$

Difference rule:

If $f(x) = u(x) - v(x)$, then $f'(x) = u'(x) - v'(x)$

Product rule:

If $f(x) = u(x) \cdot v(x)$, then $f'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$

Quotient rule:

If $f(x) = \frac{u(x)}{v(x)}$, then $f'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{\left(v(x)\right)^2}$

Power (Chain) rule:

If $f(x) = \left(u(x)\right)^n$, then $f'(x) = n\left(u(x)\right)^{n-1} \cdot u'(x)$

Examples: Find the derivative of the following functions:

1- $f(x) = 5 \Rightarrow \frac{d}{dx} f(x) = \frac{d}{dx}(5) = 0$

2- $f(x) = x^6 \Rightarrow \frac{d}{dx} f(x) = \frac{d}{dx}(x^6) = 6x^5$

3- $f(x) = x^6 \mp x^3 \Rightarrow \frac{d}{dx}(x^6 \mp x^3) = \frac{d}{dx}(x^6) \mp \frac{d}{dx}(x^3) = 6x^5 \mp 3x^2$

4- $f(x) = 3x^5 \Rightarrow \frac{d}{dx} f(x) = \frac{d}{dx}(3x^5) = 15x^4$

5- $f(x) = (x^2+2)(x^3+3x+1) \Rightarrow$

$\frac{d}{dx} f(x) = (x^2+2)(3x^2+3) + (x^3+3x+1).2x$

6- $f(x) = (x^3 - \frac{x}{2})^6 \Rightarrow \frac{d}{dx} f(x) = 6\left(x^3 - \frac{x}{2}\right)^5 (3x^2 - \frac{1}{2})$

7- $f(x) = \frac{x^2-1}{x^4+1} \Rightarrow \frac{d}{dx}\left(\frac{x^2-1}{x^4+1}\right) = \frac{(x^4+1).\frac{d}{dx}(x^2-1) - (x^2-1).\frac{d}{dx}(x^4+1)}{(x^4+1)^2}$

$$= \frac{(x^4+1).(2x) - [(x^2-1).4x^3]}{(x^4+1)^2} = \frac{2x^5+2x-[4x^5 - 4x^3]}{(x^4+1)^2}$$

$$= \frac{2x^5+2x-4x^5+4x^3]}{(x^4+1)^2} = \frac{-2x^5+4x^3+2x}{(x^4+1)^2}$$

8- $f(x) = (2x^2 - 5x^{-2})^{-5} \Rightarrow$

$\frac{d}{dx} f(x) = -5(2x^2 - 5x^{-2})^{-6}.(4x + \quad 10\, x^{-3}$

Derivative of trigonometric function

1-if $f(x) = \sin x \Rightarrow f'(x) = \cos x$

2-if $f(x) = \cos x \Rightarrow f'(x) = -\sin x$

3-if $f(x) = \tan x \Rightarrow f'(x) = sec^2\, x$

4-if $f(x) = \cot x \Rightarrow f'(x) = -csc^2\, x$

5-if $f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$

6-if $f(x) = \csc x \Rightarrow f'(x) = -\csc x\, cotx$

7-if $f(x) = e^x \Rightarrow f'(x) = e^x$

8-if $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x} \,.f'(x)$

Note: we use the following rules when $u(x)$ is a function of x:

1- $\frac{d}{dx}(\sin u(x)) = \cos u(x).u'(x)$

2- $\frac{d}{dx}(\cos u(x)) = -\sin u(x).u'(x)$

$3-\frac{d}{dx}(\tan u(x)) = sec^2 u(x).u'(x)$

4- $\frac{d}{dx}(\cot u(x)) = -csc^2 u(x).u'(x)$

5- $\frac{d}{dx}(\sec u(x)) = \sec u(x) \tan u(x).u'(x)$

6- $\frac{d}{dx}(\csc u(x)) = -cscu(x) \cot(u(x)).u'(x)$

Examples: Find the derivative of the following functions:

1- $f(x) = \sin x^3 \Rightarrow f'(x) = \cos x^3 . 3x^2 = 3x^2 \cos x^3$
2- $f(x) = \cos(5x^2+2)^3 + \cot(1-x^2) \Rightarrow f'(x) = -sin(5x^2+2)^3.3(5x^2+2)^2.10x - \csc(1-x^2)\cot(1-x^2).-2x$

3- $f(x) = \tan(x^2+1) \Rightarrow f'(x) = sec^2(x^2+1).2x$

4- $\boldsymbol{f(x) = (3-\sec(5x))^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{2}(3-\sec(5x))^{-\frac{1}{2}}.\sec(5x)\tan(5x).5}$

5- $f(x) = csc^2(2x) + 4x^2\, sinx$

$$f'(x) = 2\csc(2x).-\csc(2x)\cot(2x).2 + 4(x^2\, cosx + sinx.2x)$$

$$f'(x) = -2csc^2(2x)\cot(2x).2 + 4(x^2\, cosx + 2x\, sinx)$$

1- $= \ln(x^2-8) \Rightarrow f'(x) = \frac{1}{x^2-8}.2x = \frac{2x}{x^2-8}$
2- $f(x) = e^{-2x} \Rightarrow f'(x) = -2e^{-2x}$

H.W: Find the derivative of the following functions:

1- $\boldsymbol{f(x)} = tan^3(\mathrm{COS}\, 5x^3)^{\frac{1}{2}}$ **2-** $\boldsymbol{f(x)} = \cos(x^4 + \mathbf{3}x^3 + x^2)^{\mathbf{7}} + \csc(\tan x^3)$

3- $f(x) = \frac{(sin\sqrt{x})^3}{\sqrt{x}}$ 4- $f(x) = x^3\sin(2x^2+3)$