Q1/ Using Mathematical induction to prove that $\frac{1}{2}+\frac{1}{6}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$.

البرهان: لتكن S مجموعة جزئية تحقق العلاقة اعلاه، $S \subseteq IN$

1- $1 \in S \Leftarrow$

$$\frac{1}{2}+\frac{1}{6}+\cdots+\frac{1}{1(1+1)}=\frac{1}{1+1}$$

$$\frac{1}{2}=\frac{1}{2}$$

2- $K \in S \Leftarrow$

$$\frac{1}{2}+\frac{1}{6}+\cdots+\frac{1}{K(K+1)}=\frac{K}{K+1}$$

3- نريد ان نثبت ان $K+1 \in S$

نضيف $\frac{1}{(K+1)(K+2)}$ للطرفين

$$\frac{1}{2}+\frac{1}{6}+\cdots+\frac{1}{K(K+1)}+\frac{1}{(K+1)(K+2)}=\frac{K}{K+1}+\frac{1}{(K+1)(K+2)}$$

$$\frac{1}{2}+\frac{1}{6}+\cdots+\frac{1}{K(K+1)}+\frac{1}{(K+1)(K+2)}=\frac{1}{K+1}\left(K+\frac{1}{K+2}\right)$$

$$=\frac{1}{K+1}\left(\frac{K^2+2K+1}{(K+2)}\right)$$

$$=\frac{1}{\cancel{K+1}}\left(\frac{\cancel{(K+1)}(K+1)}{K+2}\right)=\frac{K+1}{K+2}$$

$\therefore K+1 \in S \Longrightarrow S=IN$

Q2/ Give example for

① semigroup. (شبه زمرة)

Ex!. $(\mathbb{Z}, +)$, since $\forall\, a, b, c \in \mathbb{Z}$

$(a+b)+c = a+(b+c)$ تجميعية

$\therefore (\mathbb{Z}, +)$ is semigroup

② Subset of group also is group.

كل مجموعة جزئية من زمرة ايضاً زمرة.

Ex!. $2\mathbb{Z} \subseteq \mathbb{Z}$

$(2\mathbb{Z}, +)$ is group

(راجع الطالب في المحاضرات)

③ Group with identity element.

Ex!. $(\mathbb{R} - \{0\}, \cdot)$

$\forall\, a \in \mathbb{R} \Longrightarrow a^{-1} \in \mathbb{R}$

$a \cdot a^{-1} = a^{-1} \cdot a = 1$

④ Not Group

Ex!. $(\mathbb{Z}, \div)$ $\forall\, a, b \in \mathbb{Z} \Longrightarrow \frac{a}{b} \notin \mathbb{Z}$.