**INITIAL VALUE PROBLEMS**

1. **Euler methods**

Euler's method is a numerical method to solve first order first degree differential equation with a given initial value. It is the most basic [explicit method](https://en.wikipedia.org/wiki/Explicit_and_implicit_methods) for [numerical integration of ordinary differential equations](https://en.wikipedia.org/wiki/Numerical_ordinary_differential_equations) and is the simplest [Runge–Kutta method](https://en.wikipedia.org/wiki/Runge%E2%80%93Kutta_method). The Euler method is named after [Leonhard Euler](https://en.wikipedia.org/wiki/Leonhard_Euler), who treated it in his book [*Institutionum calculi integralis*](https://en.wikipedia.org/wiki/Institutionum_calculi_integralis)

The Euler method is a first-order method, which means that the local error (error per step) is proportional to the square of the step size, and the global error (error at a given time) is proportional to the step size. The Euler method often serves as the basis to construct more complex methods, e.g., [predictor–corrector method](https://en.wikipedia.org/wiki/Predictor%E2%80%93corrector_method).

في [الرياضيات](https://ar.wikipedia.org/wiki/%D8%A7%D9%84%D8%B1%D9%8A%D8%A7%D8%B6%D9%8A%D8%A7%D8%AA) وفي [الحوسبة العلمية](https://ar.wikipedia.org/wiki/%D8%AD%D9%88%D8%B3%D8%A8%D8%A9_%D8%B9%D9%84%D9%85%D9%8A%D8%A9)، **طريقة أويلر** ([بالإنجليزية](https://ar.wikipedia.org/wiki/%D9%84%D8%BA%D8%A9_%D8%A5%D9%86%D8%AC%D9%84%D9%8A%D8%B2%D9%8A%D8%A9): Euler method) هي طريقة تعتمد أساسا على الحساب وتمكن من حلحلة [المعادلات التفاضلية العادية](https://ar.wikipedia.org/wiki/%D9%85%D8%B9%D8%A7%D8%AF%D9%84%D8%A9_%D8%AA%D9%81%D8%A7%D8%B6%D9%84%D9%8A%D8%A9_%D8%B9%D8%A7%D8%AF%D9%8A%D8%A9) من الدرجة الأولى انطلاقا من قيمة بدأية.[[1]](https://ar.wikipedia.org/wiki/%D8%B7%D8%B1%D9%8A%D9%82%D8%A9_%D8%A3%D9%88%D9%8A%D9%84%D8%B1#cite_note-1)

سميت هذه الطريقة هكذا نسبة إلى عالم الرياضيات [ليونهارد أويلر](https://ar.wikipedia.org/wiki/%D9%84%D9%8A%D9%88%D9%86%D9%87%D8%A7%D8%B1%D8%AF_%D8%A3%D9%88%D9%8A%D9%84%D8%B1) الذي درسها ي كتاب له نشر بين عامي 1768 و1770.

The Euler forward scheme may be very easy to implement but it can't give accurate solutions.    A  very small step size is required for any meaningful result.  In this scheme, since, the starting point of each sub-interval is used to find the slope of the solution curve,  the solution would be correct only if the function is linear. So an improvement over this is to take the arithmetic average of the slopes at **ti**  and **ti+1**(that is, at the end points of each sub-interval). The scheme so obtained is called modified Euler's method. It works first by approximating a value to **yi+1** and then improving it by making use of average slope.

**Consider the forward difference approximation for first derivative**



* **Rewriting the above equation we have**



* **So,**  **is recursively calculated as**



**Example:** find y(0.5)



**Solution:**





Example2:- let  and ,h=1,y(0)=1,find y(4)?

**MATLAB code example**

clear; clc; close('all');

y0 = 1;

t0 = 0;

h = 1; *% try: h = 0.01*

n = 4;

[t, y] = Euler(t0, y0, h, n);

plot(t, y, 'b');

*% exact solution (y = e^t):*

tt = (t0:0.001:n)';

yy = exp(tt);

hold('on');

plot(tt, yy, 'r');

hold('off');

legend('Euler', 'Exact');

**function** [t, y] = Euler(t0, y0, h, n)

fprintf('%10s%10s%10s%15s\n', 'i', 'yi', 'ti', 'f(yi,ti)');

fprintf('%10d%+10.2f%+10.2f%+15.2f\n', 0, y0, t0, f(y0,t0));

t = (t0:h:n)';

y = zeros(size(t));

y(1) = y0;

**for** i = 1:1:length(t)-1

y(i+1) = y(i) + h\*f(y(i),t(i));

fprintf('%10d%+10.2f%+10.2f%+15.2f\n', i, y(i+1), t(i+1), f(y(i+1),t(i+1)));

**end**

**end**

*% in this case, f(y,t) = f(y)*

**function** dydt = f(y,t)

dydt = y;

**end**

*% OUTPUT:*

*% i yi ti f(yi,ti)*

*% 0 +1.00 +0.00 +1.00*

*% 1 +2.00 +1.00 +2.00*

*% 2 +4.00 +2.00 +4.00*

*% 3 +8.00 +3.00 +8.00*

*% 4 +16.00 +4.00 +16.00*

*% NOTE: Code also outputs a comparison plot*

First two steps of Eulerâs method wit h=1  for the equation  with initial conditions y(0) = 1 .find y(2).

Solution :-

We first find xi







المصادر

* *Atkinson, Kendall A. (1989). An Introduction to Numerical Analysis (2nd ed.). New York:*[*John Wiley & Sons*](https://en.wikipedia.org/wiki/John_Wiley_%26_Sons)*.*[*ISBN*](https://en.wikipedia.org/wiki/International_Standard_Book_Number)[*978-0-471-50023-0*](https://en.wikipedia.org/wiki/Special:BookSources/978-0-471-50023-0)*.*
* *Ascher, Uri M.;*[*Petzold, Linda R.*](https://en.wikipedia.org/wiki/Linda_Petzold)*(1998). Computer Methods for Ordinary Differential Equations and Differential-Algebraic Equations. Philadelphia:*[*Society for Industrial and Applied Mathematics*](https://en.wikipedia.org/wiki/Society_for_Industrial_and_Applied_Mathematics)*.*[*ISBN*](https://en.wikipedia.org/wiki/International_Standard_Book_Number)[*978-0-89871-412-8*](https://en.wikipedia.org/wiki/Special:BookSources/978-0-89871-412-8)*.*
* [*Butcher, John C.*](https://en.wikipedia.org/wiki/John_C._Butcher)*(2003). Numerical Methods for Ordinary Differential Equations. New York:*[*John Wiley & Sons*](https://en.wikipedia.org/wiki/John_Wiley_%26_Sons)*.*[*ISBN*](https://en.wikipedia.org/wiki/International_Standard_Book_Number)[*978-0-471-96758-3*](https://en.wikipedia.org/wiki/Special:BookSources/978-0-471-96758-3)*.*
* *Lakoba, Taras I. (2012),*[*Simple Euler method and its modifications*](http://www.cems.uvm.edu/~lakobati/math337/notes_1.pdf)*(PDF) (Lecture notes for MATH334), University of Vermont, retrieved 29 February 2012*