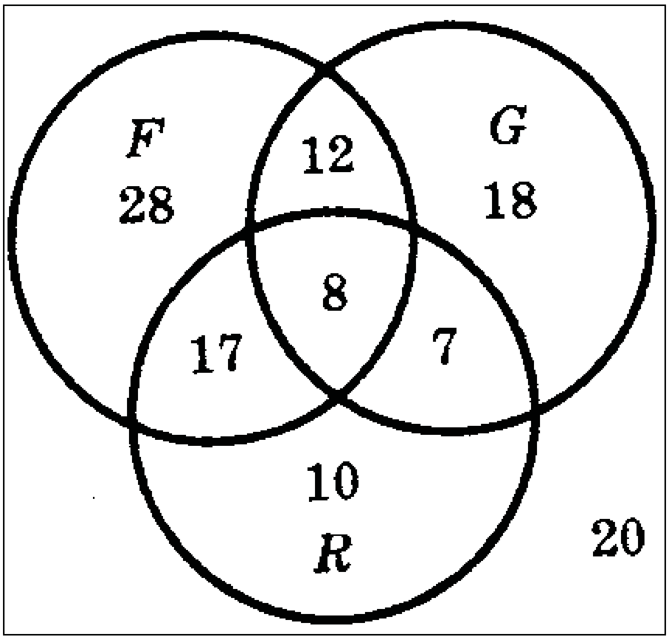


Solution:

$|F \cup G \cup R| = |F| + |G| + |R| - |F \cap G| - |F \cap R| - |G \cap R| + |F \cap G \cap R|$

$100 = 65 + 45 + 42 - 20 - 25 - 15 + |F \cap G \cap R|$

$100 = 92 + |F \cap G \cap R|$

$\therefore |F \cap G \cap R| = 8$ students study the 3 languages

$20 – 8 = 12$ $(F \cap G) - R$

$25 – 8 = 17$ $(F \cap R) - G$

$15 – 8 = 7$ $(G \cap R) - F$

65 – 12 – 8 – 17 = 28 students study French only

45 – 12 – 8 7 = 18 students study German only

42 – 17 – 8 7 = 10 students study Russian only

120 – 100 = 20 students do not study any language

Relations

Binary relation:

There are many relations in mathematics :"less than" , "is parallel to ","is a subset of", etc. These relations consider the existence or nonexistence of a certain connection between pairs of objects taken in a definite order. We define a relation simply in terms of ordered pairs of objects.

Product sets:

Consider two arbitrary sets A and B. The set of all ordered pairs (a,b) where $a \in A$ and $b \in B$ is called the product, or cartesian product, of A and B.

$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$

Example: Let A = {1,2} and B = {a ,b ,c} then

$A \times B = \{(1,a), (1,b),(1,c),(2,a),(2,b),(2,c)\}$

Also, $A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

- The order in which the sets are considered is important, so $A \times B \neq B \times A$.
Let A and B be sets. A binary relation, R, from A to B is a subset of A×B. If $(x,y) \in R$, we say that x is R-related to y and denote this by xRy
if)x,y) $\notin R$, we write x
y and say that x is not R-related to y .
if R is a relation from A to A ,i.e. R is a subset of A × A, then we say that R is a relation on A.
The **domain** of a relation R is the set of all first elements of the ordered pairs which belong to R, and the **range** of R is the set of second elements.

Example 1:
Let A = {1, 2, 3, 4}. Define a relation R on A by writing $(x, y) \in R$ if $x < y$. Then
R = {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}.

Example 2:
let A = {1,2,3} and R = {(1,2),(1,3),(3,2)}. Then R is a relation on A since it is a subset of A×A with respect to this relation:
1R2, 1R3, 3R2 but $(1,1) \notin R$ & $(2,1) \notin R$
The domain of R is {1,3} and
The range of R is {2,3}

Example 3:
Let A = {1, 2, 3}. Define a relation R on A by writing $(x, y) \in R$, such that $a \geq b$, list the element of R
$aRb \leftrightarrow a \geq b$, $a,b \in A$
$\therefore$ R = {(1,1),(2,1), (2,2), (3,1), (3,2), (3,3)}.

Example 4:
A relation on the set Z of integers is "m divides n." A common notation for this relation is to write m|n when m divides n. Thus 6 | 30 but 7*25.

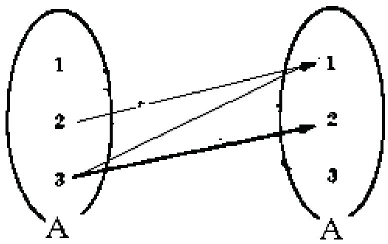
Representation of relations:
1) By language
2) By ordered pairs
3) By arrow form
4) By matrix form
5) By coordinates
6) By graph form

Example:
Let A = {1,2,3}, the relation R on A such that: $aRb \leftrightarrow a>b$; $a,b \in A$

1) By language:
R={(a,b) : $a,b \in A$ and $aRb \leftrightarrow a>b$}

2) By ordered pairs
R = {(2,1),(3,1),(3,2)}

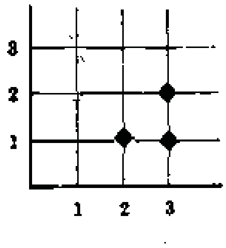
3) By arrow form



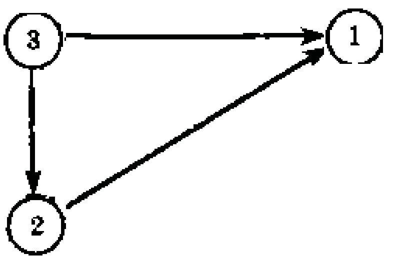
4) By matrix form

	1	2	3
1	0	0	0
2	1	0	0
3	1	1	0

5) By coordinates



6) By graph form



TYPES OF RELATIONS:
Properties of relations:

Let R be a relation on the set A

1) Reflexive : R is reflexive if : $\forall$ a $\in$A $\rightarrow$ aRa or (a,a) $\in$ R ; $\forall$ a, b $\in$A. . Thus R is not reflexive if there exists a $\in$ A such that (a, a) $\notin$ R.

2) Symmetric : aRb $\rightarrow$ bRa $\forall$ a,b $\in$A. if whenever *(a, b)* $\in$ *R* then *(b, a)* $\in$ *R*.
Thus *R* is not symmetric if there exists *a, b* $\in$ *A* such that *(a, b)* $\in$ *R* but *(b, a)* $\notin$ *R*.

3) Transitive : aRb $\wedge$ bRc $\rightarrow$ aRc. that is, if whenever *(a, b), (b, c)* $\in$ *R*
then *(a, c)* $\in$ *R*. Thus *R* is not transitive if there exist *a, b, c* $\in$ *R* such that *(a, b), (b, c)* $\in$ *R* but *(a, c)* $\notin$ *R*.

4) Equivalence relation : it is Reflexive & Symmetric & Transitive. That is, *R* is an equivalence relation on *S* if it has the following three properties:
a - For every *a* $\in$*S*, *aRa*.
b- If *aRb*, then *bRa*.
c- If *aRb* and *bRc*, then *aRc*.

5) Irreflexive : $\forall$ a $\in$A (a,a) $\notin$ R

6) AntiSymmetric : if aRb and bRa $\rightarrow$ a=b
the relations $\geq$,$\leq$ and $\subseteq$ are antisymmetric

Example 5:
Consider the relation of C of set inclusion on any collection of sets:
1) A $\subset$ A for any set, so $\subset$ is reflexive
2) A $\subset$ B dose not imply B $\subset$ A, so $\subset$ is not symmetric
3) If A $\subset$ B and B $\subset$ C then A $\subset$ C, so $\subset$ is transitive
4) $\subset$ is reflexive, not symmetric & transitive, so $\subset$ is not equivalence relations
5) A $\subset$ A, so $\subset$ is not Irreflexive
6) If A $\subset$ B and B $\subset$ A then A = B, so $\subset$ is anti-symmetric

Example 6:
If A ={1,2,3} and R={(1,1),(1,2),(2,1),(2,3)} Is R equivalence relation ?
1) 2 is in A but (2,2) $\notin$ R, so R is not reflexive
2) (2,3) $\in$ R but (3,2) $\notin$ R, so R is not symmetric
3) (1,2) $\in$ R and (2,3) $\in$ R but (1,3) $\notin$ R, so R is not transitive
So R is not Equivalence relation
Example 7 :
What is the properties of the relation =?
1) a=a for any element a $\in$ A, so = is reflexive
2) If a = b then b = a, so = is symmetric
3) If a = b and b = c then a = c, so = is transitive
4) = is (reflexive + symmetric + transitive), so = is equivalence
5) a = a, so = is not Irreflexive
6) If a = b and b = a then a = b, so = is anti-symmetric

Remark: