Home work

Q.1: Prove or disprove (choose 2)

(1) The function $f(x) = x^2-4$ is continuous at c=2**.**
(2) Every bounded sequence is convergent.
(3) The limit of convergent sequence is unique.
(4) There is one pt. of discontinuity in function
(5) $|a \mp b| \leq |a|+|b|$ for any a,b$\in R$.
(6) $f(x) = \begin{cases} -x & if\ x < -1 \\ 1 & if -1 \leq x \leq 1 \\ x & if\ x > 1 \end{cases}$ is continuous at x=1 and x =-1.
(7) The sequence $\langle 4 + \frac{(-1)^n}{n} \rangle$ is convergent to 0.
(8) The sequence $\langle (-1)^n . n \rangle$ is neither bounded above nor bound below.
(9) $\langle (-1)^n . n \rangle$ is convergent.
(10) $< \frac{n}{n^2+1} >$ is unbounded sequence.
(11) $\langle cos \frac{n\pi}{2} \rangle = \langle 0, -1, 0, 1, 0, -1, 0, 1, \ldots \rangle$ is bounded but not convergent.
(12) if f and g continuous function then f /g is continuous.

Q.2: The functions sin(x) and cos(x) are continuous.

proof.

بالنسبة لل sin(x) برهن في المحاضرات

وبنفس الطريقة يبرهن ال cos(x)

وباستخدام الحقائق المعروفة للجيب والجيب تمام وهي:

$|\sin(x)| \leq |x|,$

$|\cos(x)| \leq 1, \text{ and } |\sin(x)| \leq 1.$

$$
\begin{aligned}
|\sin(x)-\sin(c)| &= \left|2\sin\left(\frac{x-c}{2}\right)\cos\left(\frac{x+c}{2}\right)\right| \\
&= 2\left|\sin\left(\frac{x-c}{2}\right)\right|\left|\cos\left(\frac{x+c}{2}\right)\right| \\
&\le 2\left|\sin\left(\frac{x-c}{2}\right)\right| \\
&\le 2\left|\frac{x-c}{2}\right| = |x-c|
\end{aligned}
$$

$$
\begin{aligned}
|\cos(x)-\cos(c)| &= \left|-2\sin\left(\frac{x-c}{2}\right)\sin\left(\frac{x+c}{2}\right)\right| \\
&= 2\left|\sin\left(\frac{x-c}{2}\right)\right|\left|\sin\left(\frac{x+c}{2}\right)\right| \\
&\le 2\left|\sin\left(\frac{x-c}{2}\right)\right| \\
&\le 2\left|\frac{x-c}{2}\right| = |x-c|
\end{aligned}
$$

يستخدم في الحل وكما في سؤال استمرارية الجيب

Q.3: Prove that the sequence is convergent,

(1) $<\sqrt{n+1}-\sqrt{n}>\to 0$ (اضرب بمرافق المقام)

(2) prove that

the sequence $\left\{\frac{n^2+1}{n^2+n}\right\}$ converges and

$$\lim_{n\to\infty}\frac{n^2+1}{n^2+n}=1.$$

Proof. Given >0 , find M$\in N$ such that, if n$\geq M$ we have

$$
\begin{aligned}
\left|\frac{n^2+1}{n^2+n}-1\right| = \left|\frac{n^2+1-(n^2+n)}{n^2+n}\right| &= \left|\frac{1-n}{n^2+n}\right| \\
&= \frac{n-1}{n^2+n} \\
&\le \frac{n}{n^2+n} = \frac{1}{n+1} \\
&\le \frac{1}{M+1} < \varepsilon.
\end{aligned}
$$

But $\frac{1}{M+1}<\frac{1}{M}$, so $\left|\frac{n^2+1}{n^2+n}-1\right|<\varepsilon$ if $\frac{1}{M+1}<\frac{1}{M}<\varepsilon$

so $\frac{1}{M}<\varepsilon$ and M$>\frac{1}{\varepsilon}$ this means that $\frac{n^2+1}{n^2+n}\to 1$