**Eigenvalues and Eigenvectors**

Given a square matrix A, suppose there are a constant and a nonzero vector x such that

then is called an Eigenvalue of A, and is an Eigenvector of A corresponding to. The eigenvalues and the corresponding eigenvectors always exist for any given square matrix.

**Defiition1**

***Let be a matrix. There is a number and a vector such that We say that is an Eigenvalue of A, and is an Eigenvector of A****.*

**Example 1**

then

Where

The last linear system has a non-trivial solution if and only if

det

So that

To find the eigenvector , for we have to solve the following system of equations:

We have only one equation with two unknowns.

then

let then

.

Similarly, we can show that the eigenvector for is

**Defiition2**

***The equation , called the characteristic equation of a square matrix***

**Defiition1**

***The eigenvalues of a square matrix are the roots of the characteristic***

***equation*** *.*

**Example 1**

Find Eigenvalue of and Eigenvector of the matrix

*Solution*

the characteristic equation is det

The eigenvalues are, therefore, r = −1 and 6.

, or

Next, we will substitute each of the two eigenvalues into the matrix equation . For , the system of linear equations is

Notice that the matrix equation represents a degenerated system of two linear equations. Both equations are constant multiples of the equation

There is now only one equation for the 2 unknowns, therefore, there are infinitely many possible solutions. This is always the case when solving for eigenvectors. Necessarily, there are infinitely many eigenvectors corresponding to each eigenvalue.

Solving the equation

we get the relation

Hence, the eigenvectors corresponding to are all nonzero multiples of

.

Similarly , the eigenvectors corresponding to are all nonzero multiples of

.

**Note**

If A is any 2 × 2 matrix, then its characteristic equation is

.

*So that*

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