

Q.1 Show that the mathematical system $(G, *)$ is group

$$\exists G = \mathbb{Z} \text{ and } * : a * b = 2a + 2b - 5 \quad \forall a, b \in \mathbb{Z}$$

Solution : ① closure

$$\forall a, b \in \mathbb{Z} \Rightarrow a * b = 2a + 2b - 5$$

\therefore closure is satisfy

② Associative

$$\text{let } a, b, c \in \mathbb{Z}, \text{ t.p. } (a * b) * c = a * (b * c)$$

$$\text{L.H. } (a * b) * c =$$

$$\begin{aligned} (2a + 2b - 5) * c &= 2(2a + 2b - 5) + 2c - 5 \\ &= 4a + 4b + 2c - 15 \quad \dots \textcircled{1} \end{aligned}$$

$$\text{R.H. } a * (b * c) = a * (2b + 2c - 5)$$

$$= 2a + 2(2b + 2c - 5) - 5$$

$$= 2a + 4b + 4c - 15 \quad \dots \textcircled{2}$$

$$\therefore \textcircled{1} \neq \textcircled{2}$$

\therefore Associative is not satisfy

$\therefore (G, *)$ is not group