Q,/ Show that the mathematical system $(R, \oplus)$ is gp.

$\exists \oplus : a \oplus b = a+b-4 \quad \forall a,b \in R$

Solution : ① closure

$\forall a,b \in R \Rightarrow a \oplus b = a+b-4 \in R$

∴ closure is satisfy

② Associative

let $a,b,c \in R$, t.p. $(a*b)*c = a*(b*c)$

L.H. $(a*b)*c = (a+b-4)*c$

$= a+b+c-8 \quad \cdots ①$

R.H. $a*(b*c) = a*(b+c-4)$

$= a+b+c-8 \quad \cdots ②$

∴ ① = ②

∴ Associative is satisfy

③ Identity

$a*e = e*a = a$

$a+e-4 = a \Rightarrow \boxed{e=4}$

④ $\forall a \in R, \exists a^{-1} \in R \quad \exists a * a^{-1} = a^{-1} * a = e$

$a * a^{-1} = e$

$a + a^{-1} - 4 = 4 \Rightarrow a^{-1} = 4 + 4 - a \Rightarrow \boxed{a^{-1} = 8 - a}$

∴ $(R, \oplus)$ is group