

Q2/ In a group  $(G, *) \ni G = \mathbb{R} \times \mathbb{R}$  and

$*$   $(a, b) * (c, d) = (a+c, b+d+2bd)$ . Find identity and inverse elements.

Solution: ① Identity element

$$\text{let } e = (c, d) \Rightarrow a * e = e * a = a$$
$$(a, b) * (c, d) = (a, b)$$

$$(a+c, b+d+2bd) = (a, b)$$

$$\hat{=} a+c = a \Rightarrow c = 0$$

$$b+d+2bd = b$$

$$d(1+2b) = 0 \Rightarrow d = 0$$

$$\hat{=} e = (c, d) = (0, 0)$$

② Inverse element

$$\forall (a, b) \in \mathbb{R} \times \mathbb{R}, \exists (a, b)^{-1} \in \mathbb{R} \times \mathbb{R} \ni$$
$$\parallel$$
$$(c, d)$$

$$a * a^{-1} = a^{-1} * a = e$$

$$(a, b) * (c, d) = (0, 0)$$

$$(a+c, b+d+2bd) = (0, 0)$$

$$a+c = 0 \Rightarrow c = -a$$

$$b+d+2bd = 0$$

$$d(1+2b) = -b \Rightarrow d = \frac{-b}{1+2b}$$

$$\hat{=} a^{-1} = (c, d) = \left(-a, \frac{-b}{1+2b}\right)$$