Chapter One
Introduction

Numerical Analysis is a field of mathematics that concerned with the study of approximate solutions of mathematical problems, where it is difficult or impossible to find the exact solutions for these problems.

For instance, we can't find the exact value of the following integral

, by using known integration methods, $\int_0^1 e^{x^2}\,dx$

while we can find an approximate value for this integral, using numerical methods.

The other example, if we aim to find the solution of a linear system $Ax = b,$ where $A \in R^{n\times n}$, when n is too large , it so difficult to find the exact solutions for this system by hand using known methods, so in this case, it is easier to think about how to find the approximate solution using a suitable algorithm and computer programs.

The importance of Numerical Analysis

To interpret any real phenomena, we need to formulate it, in a mathematical form. To give a realistic meaning for these phenomena we have to choose complicated mathematical models, but the problem is, it so difficult to find explicit formulas to find the exact solutions for these complicated models. Therefore, it might be better to find the numerical solutions for complicated form rather than finding the exact solutions for easier forms that can't describe these phenomena in realistic way.

The nature of Numerical analysis

Since for any numerical Algorithm (the steps of the numerical method), we have lots of mathematical calculations, we need to choose a suitable computer language such as Matlab or Mable and write the algorithm processes in programing steps.

In fact, the accuracy of numerical solutions, for any problem, is controlled by three criteria:

1- The type of algorithm,

2- The type of computer language and programs,

3- The advancement of the computers which are used.

Types and sources of Errors

When we compute the numerical solutions of mathematical model that we use to describe a real phenomenon, we get some errors; therefore we should study the types and sources of these errors.

We can point out the most important types and sources of these errors as follows:

1- *Rounding errors*: this type of errors can be got, because of the rounding of numbers in computer programming languages.

Example:- 5.99…9 round to 6, and 3.0001 round to 3.

2- *Truncation Errors:* Since most of numerical algorithms depend on writing the functions as infinite series, and since it's impossible to take more than few terms of these series when we formulate the algorithm, therefore, we get errors, called truncation errors.

Example :-

$$f(x) = e^{x^2} = 1 + x + x^2/2! + x^3/3! + \cdots$$

If we compute $f(5)$, with taking 4 terms, we get more errors than with taking 10 terms.

3-*Total errors*:- Since any numerical algorithm is about iterative presses, the solution in step n depends on the solution in step $(n-1)$. Therefore, for larger number of steps we get more errors, and those errors are the total of all previous types of errors.

Let $\bar{x}$ is the approximate value of x, there are two methods can be used to measure the errors:-

1- Absolute Error:- $\boldsymbol{E_x = |x - \bar{x}\,|}$,

2- Relative error:- $\boldsymbol{R_x = \frac{E_x}{|x|} = \frac{|x-\bar{x}\,|}{|x|}}$

Example:- Let $\bar{x} = 3.14$, $x = 3.141592$. Find the Absolute and Relative errors

Solution

$E_x = |x - \bar{x}\,| = 0.001592$,

$$R_x = \frac{0.001592}{3.141592} = 0.000507$$

Remark: Clearly, $R_x < E_x$

Questions

Q1:

i- What is numerical analysis concerned with?, and what is the importance of studying this subject ?,

ii- What are the most important types of errors that arise from using a numerical method to compute the numerical solution of a mathematical problem?

iii- What are the criteria that control the accuracy of numerical solutions?

Q2: Let E_x, R_x be the absolute and relative errors, respectively, in an approximate value of x . Show that $R_x \leq E_x$, if $|x| \geq 1$.