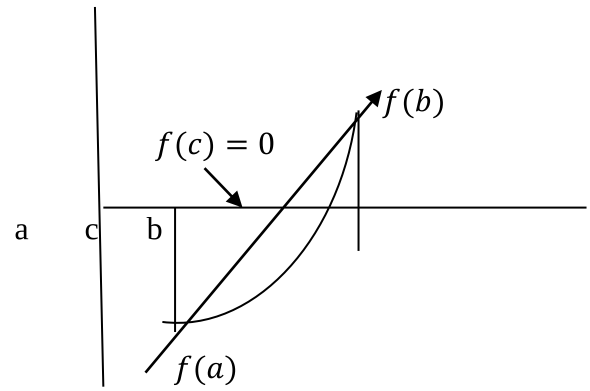
2. False position Algorithm

This algorithm is faster than Bisection algorithm in convergence, but they almost smaller in steps. In fact, it has been called by this name, because in each iterative step we find an approximate position for the exact root, and we assume that this place is the real position of the exact root, which means we assume that the equation equal zero at this position.
Before starting in the steps of this algorithm, we have to make sure that:
$f \in C[a,b]$, and $f(a)f(b) < 0$
The main idea of this method is to assume that f is linear on $[a,b]$, which means the point $(c,0)$ belongs to the line AB
where $A = (a, f(a))$, $B = (b, f(b))$.



Since the two points $(c,0), (b, f(b))$, belong to the same line, we have

$$\frac{\mathrm{f(b) - f(a)}}{\mathrm{b - a}} = \frac{\mathrm{f(b) - 0}}{\mathrm{b - c}}$$

Thus

$$c = b - f(b)\,\left(\frac{b-a}{f(b)-f(a)}\right) = \frac{af(b) - bf(a)}{f(b) - f(a)}$$

Algorithm steps

1-Find c by using the relation

$$c = \frac{af(b)-bf(a)}{f(b)-f(a)}$$

2- As in Bisection algorithm, we check whether, $f(c) = 0$, we know that c is the exact root, otherwise check the sign of $f(c)$, if $f(c) < 0, f(b) > 0$, then the exact root belong to [c,b].And we set $a = c, b = b$, and then we repeat the same steps.

While, if $f(c) > 0, f(a) < 0$, then the exact root belong to $[a, c]$, and we set $a = a, b = c$, and then we repeat the same steps.

3- We continue until to get the following condition is satisfied

$|c_{n+1} - c_n| < \in$

Remark :- From the iterative processes, as in the Bisection algorithm we get a sequence of approximate roots, which is convergent to the exact root

i.e. $\{c_n\} \xrightarrow{n \to \infty} \alpha$

Example:- Find the approximate roots of the following equation

$f(x) = x^2 + x - 1 = 0,$ on the interval [0,1], by using False position algorithm, for two iterative steps (find only c_1, c_2).

Solution

n	a_n	$f(an)$	b_n	$f(bn)$	c_n	$f(c_n)$
0	0	-1	1	1	0.50	-0.25
1	0.50	-0.25	1	1	0.60	-0.040