4-**Secant Algorithm**

This algorithm is faster than Bisection and False position algorithms in convergence. We use this algorithm, when it is difficult to find the derivative f', where $f(x) = 0$.

We can derive the formula of this method as follows:

Let $\{f_n\}$ is a sequence of approximate roots for f

Form the definition of derivative, we get

$$f'(p_{n-1}) = \lim_{x \to p_{n_1}} \frac{\mathrm{f(x)} - \mathrm{f}(p_{n-1})}{\mathrm{x} - p_{n-1}}$$

Set $x = p_{n-1}$, $we\ get$

$$f'(p_{n-1}) = \lim_{x \to p_{n_1}} \frac{\mathrm{f}(p_{n-2}) - \mathrm{f}(p_{n-1})}{p_{n-2} - p_{n-1}}$$

$$f'(p_{n-1}) \cong \frac{\mathrm{f}(p_{n-2}) - \mathrm{f}(p_{n-1})}{p_{n-2} - p_{n-1}} \text{..........(1)}$$

Recall N.R. equation

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})} \text{..............(2)}$$

Substitute (1) in (2), we get the secant equation

$$p_n = p_{n-1} - \frac{f(p_{n-1})(p_{n-2} - p_{n-1})}{\mathrm{f}(p_{n-2}) - \mathrm{f}(p_{n-1})}$$

Remark

Unlike to N.R. algorithm, In order to use the last equation to find the approximate roots, we need to choose two initial roots, p_0, p_1 , and that can be considered one of the disadvantage to using secant algorithm, however we do not need to find the derivative as in N.R. algorithm.

The steps of secant algorithm

1- input p_0, p_1

2- Find $p_,$using secant equation

$$p \quad = p_1 - \frac{f(p_1)(p_0 - p_1)}{\mathrm{f}(p_0) - \mathrm{f}(p_1)}$$

3- set $p_0 = p_{1,}\ \ p_1 = p$

4- we repeat the iterative presses until we get

$$|p - p_n| < \epsilon$$

5-print the last approximate root and the number of iterative processes until we get the convergence

Remark :- This algorithm has been called secant algorithm, because, the line which contains the two points

$$,\ (p_0, f(p_0)), (p_1, f(p_1))$$

intersect with the x-axis at the point $(p, 0)$.

Example:- Use the secant Algorithm, to find the approximate solution of the equation $f(x) = x^2 - 4 = 0$, assume that

$p0 = 3$, $p_1 = 2.5$, for two iterative steps.

Solution

$$p \quad = p_1 - \frac{f(p_1)(p_0 - p_1)}{\mathrm{f}(p_0) - \mathrm{f}(p_1)} = 2.0909$$

$p_0 = 2.5$, $p_1 = p = 2.0909$

$$p \quad = p_1 - \frac{f(p_1)(p_0 - p_1)}{f(p_0) - f(p_1)} = 2.0099$$

n	p_0	$f(p_0)$	p_1	$f(p_1)$	p	*f(p)*
0	3	5	2.5	2.2500	2.0909	0.3719
1	2.5	2.2500	2.0909	0.3719	2.0099	0.0397