

**Interpolation by Finite Differences**

As we have seen, the Lagrange interpolation method can be used even though, the distances between the points in the data base are not equal, for example  $X = 2,5,7,12$ . While the finite difference methods, which we will study, can be used only when the distances are equal.

For example  $X = 2,4,6,8,10$ .

Therefore, this can be considered as advantage of using Lagrange method. On the other hand, we can point out some disadvantage of using Lagrange method,

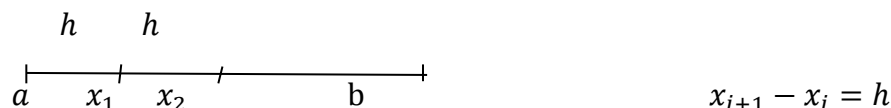
- 1-lots of mathematical operations
- 2-if we would like to add later other data, to the data base, we have to restart from the beginning, which means we waste time. While in finite difference methods, we can add any other data later.

**Finite Difference Operators**

The finite difference operators can be classified into three types:

**Forward  $\Delta$ , Backward  $\nabla$  and Center  $\delta$ .**

In order to understand the difference operators, we divide the interval  $[a, b]$ , as follows:



where  $h$ , is the distance between any two points

**Forward difference operator  $\Delta$**

The Forward difference operator is defined as follows:

$$\begin{aligned}
 \Delta y_i &= \Delta f(x_i) = f(x_i + h) - f(x_i) \\
 &= f(x_{i+1}) - f(x_i) \quad i = 0,1,2, \dots \\
 &= y_{i+1} - y_i \\
 \Delta^2 y_i &= \Delta^2 f(x_i) = \Delta(\Delta f(x_i)) = \Delta(f(x_{i+1}) - f(x_i)) = \Delta f(x_{i+1}) - \Delta f(x_i) \\
 &= f(x_{i+2}) - f(x_{i+1}) - f(x_{i+1}) + f(x_i) = y_{i+2} - 2y_{i+1} + y_i
 \end{aligned}$$

By the same way, we can find  $\Delta^3 y_i$

**Backword difference operator  $\nabla$**

The Backward difference operator is defined as follows:

$$\begin{aligned} \nabla y_i &= \nabla f(x_i) = f(x_i) - f(x_{i-1}), \quad i = 1, 2, \dots, n \\ &= y_i - y_{i-1} \\ \nabla^2 y_i &= \nabla(\nabla y_i) = \nabla(y_i - y_{i-1}) \\ &= \nabla y_i - \nabla y_{i-1} \\ &= y_i - y_{i-1} - y_{i-1} + y_{i-2} \\ &= y_i - 2y_{i-1} + y_{i-2} \end{aligned}$$

**Center difference operator  $\delta$**

The centre difference operator is defined as follows:

$$\begin{aligned} \delta y_i &= \delta f(x_i) = f\left(x_i + \frac{h}{2}\right) - f\left(x_i - \frac{h}{2}\right) = y_{i+\frac{1}{2}} - y_{i-\frac{1}{2}} \\ \delta^2 y_i &= \delta\left(y_{i+\frac{1}{2}} - y_{i-\frac{1}{2}}\right) \\ &= \delta y_{i+\frac{1}{2}} - \delta y_{i-\frac{1}{2}} \\ &= y_{i+1} - y_i - y_i + y_{i-1} \\ &= y_{i+1} - 2y_i + y_{i-1} \end{aligned}$$

**Newton’s formulas by using (Forward, Backward, Center) Finite Difference Operators**

Consider, we have the following data base:

Where ,  $x_i = x_0 + ih, \quad h = \frac{x_{i+1}-x_i}{n}$

$X$	$x_0$	$x_1$	.....	$x_n$
$Y$	$y_0$	$y_1$	.....	$y_n$

Our aim to find  $f(x_p)$ , where  $0 < p < 1, \quad x_p = x_0 + ph$

If  $x_p$  close to the beginning of the database (  $x_0 < x_p < x_1$ ), then we use Newton *Forward formula*, which takes the form:

$$f(x_p) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \dots$$

while, if  $x_p$  close to the end of the data base, ( $x_{n-1} < x_p < x_n$ ), then we use Newton *Backward formula*, which takes the form:

$$f(x_p) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \dots$$

where,  $x_p = x_n + ph$

If  $x_p$  close to the centre of the database, ( $x_i$ ), then we use Newton centre formula, which takes the form:

$$f(x_p) = py_{i+1} + \frac{(p^3 - p)}{6}\delta^2 y_{i+1} + qy_i + \frac{(q^3 - q)}{6}\delta^2 y_i$$

where  $x_p = x_i + ph$ ,  $p + q = 1$

### Steps of Newton's formulas:

1- Input the values of  $x = x_i)_{i=0}^n, y = y_i)_{i=0}^n$ ,

2- Input  $x_p$

3-set  $h = x_1 - x_0$

4- If  $x_p$  is close to  $x_0$ , and set  $p = \frac{x_p - x_0}{h}$ , compute  $\Delta^2 y_0, \Delta y_0$  and use Newton's forward formula, to find  $f(x_p)$ ,

If  $x_p$  close to  $x_n$ , set  $p = \frac{x_p - x_n}{h}$ , compute  $\nabla^2 y_0, \nabla y_0$ , and use Newton's Backward formula, to find  $f(x_p)$ ,

If  $x_i \leq x_p \leq x_{i+1}$ , where  $i \neq 1, i + 1 \neq n$ , set  $p = \frac{x_p - x_i}{h}$ , compute  $\delta^2 y_i, \delta^2 y_{i+1}$

and use Newton's center formula, to find  $f(x_p)$ .