Example:- consider the following data base

X	4	6	8	10
$f(x)$	1	3	8	20

Find the approximate value for each of the following:

1- $f(4.5)$ 2- $f(9)$ 3- $f(6.4)$

Solution

1- Since 4.5 close to the beginning of the data base, we use the forward Newton formula

$$x_p = x_0 + ph \Longrightarrow P = \frac{x_p - x_0}{h} = \frac{4.5-4}{2} = \frac{0.5}{2} = 0.25$$

$$f(x_p) = y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0$$

$$\Delta y_0 = y_1 - y_0 = 3 - 1 = 2$$

$$\Delta^2 y_0 = y_2 - 2y_1 + y_0 = 8 - 2(3) + 1 = 3$$

or for simplicity $\Delta y_0, \Delta^2 y_0$ can be also found from the following figure

x	y	Δ	Δ^2
4	1		
		$\longrightarrow 2 = \Delta y_0$	
6	3		$\longrightarrow 3 = \Delta^2 y_0$
		$\longrightarrow 5$	
8	8		$\longrightarrow 7 = \Delta^2 y_1$
		$\longrightarrow 12$	
10	20		

Thus

$$f(4.5) = 1 + \frac{1}{4}(2) + \left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)\frac{(3)}{2!} = \frac{39}{32} = 1.2188$$

2- Since the point 9 close to the end of the database $(x_3 = 10)$, we use the Backward Newton formula,

Set, $x_p = x_n + ph$

$$p = \frac{x_p - x_n}{h} = \frac{9-10}{2} = -\frac{1}{2}$$

$$f(x_p) = y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n$$

$$\nabla y_n = y_n - y_{n-1} = 20 - 8 = 12$$

$$\nabla^2 y_n = y_n - 2y_{n-1} + y_{n-2} = 20 - 2(8) + 3 = 7$$

Or for simplicity ∇y_3 , $\nabla^2 y_3$ can be also found from the following figure

x y ∇ ∇^2

4 1

$\longrightarrow$ 2

6 3 $\longrightarrow$ $3 = \nabla^2 y_2$

$\longrightarrow$ 5

8 8 $\longrightarrow$ $7 = \nabla^2 y_3$

$\longrightarrow$ $12 = \nabla y_3$

10 20

Thus $f(9) = 10 + (-\frac{1}{2})(12) + \frac{\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)}{2}(7) = 3.1250$

Finally, since 6.4 close to the middle of the database (between x_1 & x_2) , we use the Center Newton formula

$x_p = x_1 + ph$

$p = \frac{x_p - x_1}{h} = \frac{6.4-6}{2} = 0.2$, $q = 1 - p = 0.8$

$$f(x_p) = py_2 + \frac{(p^3 - p)}{6}\delta^2 y_2 + qy_1 + \frac{(q^3 - q)}{6}\delta^2 y_1$$

$$\delta^2 y_1 = y_2 - 2y_1 + y_0 = 8 - 2(3) + 1 = 3$$

$$\delta^2 y_2 = y_3 - 2y_2 + y_1 = 20 - 2(8) + 3 = 7$$

For simplicity, $\delta^2 \mathrm{y}_1, \delta^2 \mathrm{y}_2$ can be found from the following figure

x	y	δ	δ^2
4	1		
		$\longrightarrow$ 2	
6	3		$\longrightarrow$ $3 = \delta^2 y_1$
		$\longrightarrow$ 5	
8	8		$\longrightarrow$ $7 = \delta^2 y_2$
		$\longrightarrow$ 12	
10	20		

Thus

$$f(6.4) = (0.2)(8) + \frac{[(0.2)^3 - 0.2]}{6}(7) + (0.8)(3) + \frac{[(0.8)^3 - 0.8]}{6}(3)$$

=3.6320