Example:- consider the following data base

Х	4	6	8	10
f(x)	1	3	8	20

Find the approximate value for each of the following: 1- f(4.5) 2- f(9) 3- f(6.4)

Solution

1- Since 4.5 close to the beginning of the data base, we use the forward Newton formula

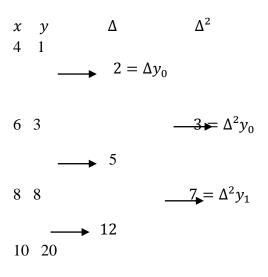
$$x_{p} = x_{0} + ph \qquad \qquad \sum P = \frac{x_{p} - x_{0}}{h} = \frac{4.5 - 4}{2} = \frac{0.5}{2} = 0.25$$

$$f(x_{p}) = y_{p} = y_{0} + p\Delta y_{0} + \frac{p(p-1)}{2!}\Delta^{2}y_{0}$$

$$\Delta y_{0} = y_{1} - y_{0} = 3 - 1 = 2$$

$$\Delta^{2}y_{0} = y_{2} - 2y_{1} + y_{0} = 8 - 2(3) + 1 = 3$$

or for simplicity Δy_0 , $\Delta^2 y_0$ can be also found from the following figure



Thus

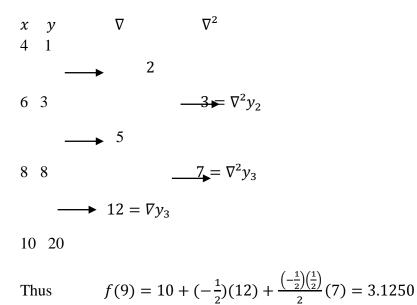
Set,

$$f(4.5) = 1 + \frac{1}{4}(2) + \left(\frac{1}{4}\right)\left(-\frac{3}{4}\right)\frac{(3)}{2!} = \frac{39}{32} = 1.2188$$

2- Since the point 9 close to the end of the database $(x_3 = 10)$, we use the Backward Newton formula,

 $x_{p} = x_{n} + ph$ $p = \frac{x_{p} - x_{n}}{h} = \frac{9 - 10}{2} = -\frac{1}{2}$ $f(x_{p}) = y_{p} = y_{n} + p\nabla y_{n} + \frac{p(p+1)}{2!}\nabla^{2}y_{n}$ $\nabla y_{n} = y_{n} - y_{n-1} = 20 - 8 = 12$ $\nabla^{2}y_{n} = y_{n} - 2y_{n-1} + y_{n-2} = 20 - 2(8) + 3 = 7$

Or for simplicity ∇y_3 , $\nabla^2 y_3$ can be also found from the following figure

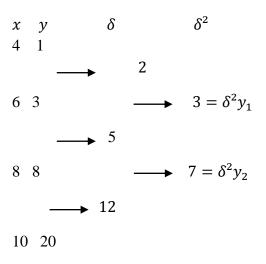


Finally, since 6.4 close to the middle of the database (between $x_1 \& x_2$), we use the Center Newton formula

$$x_p = x_1 + ph$$

$$p = \frac{x_p - x_1}{h} = \frac{6.4 - 6}{2} = 0.2$$
, $q = 1 - p = 0.8$

For simplicity, $\delta^2 y_1$, $\delta^2 y_2$ can be found from the following figure



Thus

$$f(6.4) = (0.2)(8) + \frac{[(0.2)^3 - 0.2]}{6}(7) + (0.8)(3) + \frac{[(0.8)^3 - 0.8]}{6}(3)$$

=3.6320