Interpolation by using divided finite differences

Newton's finite difference methods, which we have studied, cannot be used unless the spaces between any two points are the same. Therefore, we will study the divided finite difference, which can be used for any data base.

Consider the following data base:

x	x_0	x_1	……………	x_n
y	y_0	y_1	……………	y_n

Our aim is to find f(x_m), where $x_m \neq x_i)_{\mathrm{i=0}}^{\mathrm{n}}$, and whether x_m close or far from the beginning or the end of the data base, that does not make difference.

The general form of the Newton's equation with using divided finite difference, takes the following form:

$$f(x_m) = f(x_0) + (x_m - x_0)\mathrm{D}y_0 + (x_m - x_0)\,(x_m - x_1)\,\mathrm{D}^2y_0 + (x_m - x_0)\,(x_m - x_1)\,(x_m - x_2)\,\mathrm{D}^3y_0$$

Where $\mathrm{Dy_0}, \mathrm{D^2y_0}, \mathrm{D^3y_0}$ can be found as follows:

$\mathrm{D}y_0 = (\frac{y_1 - y_0}{x_1 - x_0})$, $\mathrm{D}y_1 = (\frac{y_2 - y_1}{x_2 - x_1})$

$\mathrm{D}^2y_0 = (\frac{\mathrm{D}y_1 - \mathrm{D}y_0}{x_2 - x_0})$, $\mathrm{D}^2y_1 = (\frac{\mathrm{D}y_2 - \mathrm{D}y_1}{x_3 - x_1})$

$$\mathrm{D}^3y_0 = (\frac{\mathrm{D}^2y_1 - \mathrm{D}^2y_0}{x_3 - x_0})$$

Steps of Newton's algorithm of divided finite difference.

1-input $\mathrm{x} = x_i)_{\mathrm{i=0}}^{\mathrm{n}}$, $\mathrm{y} = y_i)_{\mathrm{i=0,}}^{\mathrm{n}}$

2-input x_m

3-find $\mathrm{Dy_0}, \mathrm{D^2y_0}, \mathrm{D^3y_0}$

4-use Newton's formula of divided finite difference

$$f(x_m) = f(x_0) + (x_m - x_0)\mathrm{D}y_0 + (x_m - x_0)(x_m - x_1)\,\mathrm{D}^2y_0 + (x_m - x_0)(x_m - x_1)(x_m - x_2)\,\mathrm{D}^3y_0$$

Example:- Find the approximate value of f(2), from the following data base**.**

X	0	1	4	6
Y	-10	20	14	30

X Y D D^2 D^3

x_0 y_0

$\longrightarrow$ $\mathrm{D}y_0 = \left(\frac{y_1 - y_0}{x_1 - x_0}\right) = 30$

x_1 y_1 $\longrightarrow$ $\mathrm{D}^2y_0 = \left(\frac{\mathrm{D}y_1 - \mathrm{D}y_0}{x_2 - x_0}\right) = -8$

$\longrightarrow$ $\mathrm{D}y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) = -2$ $\longrightarrow$ $\mathrm{D}^3y_0 = \left(\frac{\mathrm{D}^2y_1 - \mathrm{D}^2y_0}{x_3 - x_0}\right) = \frac{10}{6}$

x_2 y_2 $\longrightarrow$ $\mathrm{D}^2y_1 = \left(\frac{\mathrm{D}y_2 - \mathrm{D}y_1}{x_3 - x_1}\right) = 2$

$\longrightarrow$ $\mathrm{D}y_2 = \left(\frac{y_3 - y_2}{x_3 - x_2}\right) = 8$

x_3 y_3

$$f(x_m) = f(x_0) + (x_m - x_0)\mathrm{D}y_0 + (x_m - x_0)(x_m - x_1)\,\mathrm{D}^2y_0 + (x_m - x_0)(x_m - x_1)(x_m - x_2)\,\mathrm{D}^3y_0$$

$$f(2) = -10 + (2-0)(30) + (2-0)(2-1)(-8) + (2-0)(2-1)(2-4)\left(\frac{10}{6}\right)$$

$$= 27.33$$