Chapter 5

Numerical integration

In this chapter, we study some methods; used to find the approximate value for the definite following integral

$$\int_a^b f(x)dx \qquad (\ \forall x \in [a,b] \ \ f \in C[a,b]) \, ,$$

when it's difficult to find the exact value by using known methods (integration methods), such as:

$$\int_a^b e^{x^2} dx$$

The general idea of the integration methods is to divide the interval $[a,b]$, into n of subintervals :

$$[a,b] = [x_0, x_1] \ \cup [x_1, x_2] \dots \dots \cup [x_{n-1}, x_n].$$

It is not needed to be the distances between the points $\{x_i\}$ are equal.

Next, we consider the polynomial p_n(x) as an approximate form for f.

$f(x) \cong p_n(x), \ \ \forall \, x \in [a,b]$

which means the problem becomes

$$\int_a^b f(x)dx \cong \int_a^b p_n(x)\, dx$$

From the last form, we note that, the formula of numerical integration depends on the way of choosing the polynomial p_n.

The general formula of integration methods takes the form:

$$\int_a^b f(x)dx \cong \int_a^b p_n(x)\, dx = \sum_{i=0}^{n} a_i \, f(xi) \dots \dots . . (*)$$

where,

$\{a_i\}_{i=0}^n$ are called the coefficients

$\{x_i\}_{i=0}^n$ are called the nodes.

If we used Lagrange polynomial, we could very easy get $\{a_i\}_{i=0}^n$, $\{x_i\}_{i=0}^n$ as follows:

We divide the interval $[a,b]$, to the n of subintervals

$$[a,b] = [x_0, x_1] \cup [x_1, x_2] \ldots \ldots \cup [x_{n-1}, x_n],$$

where the distances between the nodes $\{xi\}_{i=0}^{n}$ are equal

i.e. $x_{i+1} - x_i = h$

Thus $f(x) = p_n(x) + T(x), \ \forall\, x \in [a,b],$

where p_n is the Lagrange polynomial

$$\int_a^b f(x)dx = \int_{x_0}^{x_n} p_n(x)\,dx = \int_{x_0}^{x_n} \sum_{i=0}^{n} L_i(x) f(x_i) dx + \int_{x_0}^{x_n} \frac{f^{(n+1)}(\delta(x))}{(n+1)!} \prod_{i=0}^{n} (x - x_i) dx,$$

where $\int_{x_0}^{x_n} \frac{f^{(n+1)}(\delta(x))}{(n+1)!} \prod_{i=0}^{n} (x - x_i) dx,$ is the truncation error formula

So, $\int_a^b f(x)dx \cong \sum_{i=0}^{n} (\int_{x_0}^{x_n} L_i(x)dx) f(x_i))$ ……(1),

which means

$a_i = (\int_{x_0}^{x_n} L_i(x)dx)$

In the last formula,

For n=1, method is called (**Trapezoidal method**),

For n=2, method is called (**Simpson method**).