Chapter 6
Numerical Solutions of First Order Ordinary Differential Equations

It is well known that, an ordinary differential equation (O.D.E.), is an equation has unknown function *y*, of one variable *x*, and some of its derivatives. For instance

$$\frac{dy}{dx} = y \sin x$$

while, partial differential equation, has unknown function of two or more variables and some of its partial derivatives. For instance

$$\frac{\partial y}{\partial x} = \frac{\partial^2 y}{\partial x^2}$$

The order of the differential equation, is the highest derivative appears in the differential equation.

In this chapter, we will study, the numerical solutions for first order ordinary differential equations, which takes the general form:

$$y' = f(x, y)$$

The solution of the differential equation above, is an differentiable function defined on an interval $[a, b]$, and satisfies the differential equation. Each solution has a constant *c*, which can be determined, if *y* is known for at least one point $x_0 \in [a, b]$. ($y(x_0) = y_0$ *initial condition*)

Remark : The problem of the differential equation with an initial condition is called **Initial Value Problem:**

$$y' = f(x, y), \quad y(x_0) = y_0$$

Example:

$$y = -sinx$$
$$y(0) = 1$$

While, if y are given at more than one point, then the problem is called **Boundary values problem:**

Example

$$y = xy$$
$$y(0) = 1\,, \qquad y(1) = 2$$

In this chapter, we will only study the numerical solutions of first order initial vale problems.
Our aim is find the approximate solution, for this problem at certain points: $\{x_i\}_{i=1}^{n} \in [a, b]$, which means, we only need to find $\{y_i\}_{i=1}^{n}$