## 13. Bresenham Circle Drawing Algorithm

## Explanation:

One of the most efficient and easiest to drive of the circle algorithms is due to Bresenham. To begin, note that only one octant of the circle need be generated. The other parts can be obtained by successive reflections. If the first octant ( 0 to 45 ccw ) is generated, the second octant can be obtained by reflection through the line $\mathrm{y}=\mathrm{x}$ to yield the first quadrant. The results in the first quadrant are reflected through the line $\mathrm{x}=0$ to obtain those in the second quadrant.


Generation of a complete circle from the fist octant
The combined result in the upper semicircle are reflected through the line $\mathrm{y}=0$ to complete the circle. Bresenham's Algorithm is consider the first quadrant of an origin- centered circle. If the algorithm begins at $\mathrm{x}=0, \mathrm{y}=\mathrm{r}$, then for clockwise generation of the circle $y$ is a monotonically decreasing function
of x in the first quadrant. Here the clockwise generation starting at $x=0, y=r$ is chosen. The center of the circle is $(0,0)$.

We cannot display a continuous arc on the raster display. Instead, we have to choose the nearest pixel position to complete the arc.

From the following illustration, you can see that we have put the pixel at ( $\mathrm{X}, \mathrm{Y}$ ) location and now need to decide where to put the next pixel: at $\mathrm{N}(\mathrm{X}+1, \mathrm{Y})$ or at $\mathrm{S}(\mathrm{X}+1, \mathrm{Y}-1)$.


This can be decided by the decision parameter $\mathbf{p}$.

* If $\mathrm{p}<=0$, then $\mathrm{N}(\mathrm{X}+1, \mathrm{Y})$ is to be chosen as next pixel.
* If $\mathrm{p}>0$, then $\mathrm{S}(\mathrm{X}+1, \mathrm{Y}-1)$ is to be chosen as the next pixel.


## Algorithm

Step-1: Input radius $r$ and circle center $\left(\mathrm{X}_{\mathrm{C}}, \mathrm{Y}_{\mathrm{C}}\right)$ obtained the first point

$$
\left(\mathrm{X}_{0}, \mathrm{Y}_{0}\right)=(0, \mathrm{r})
$$

Step-2:Calculate the initial value of decision parameter as

$$
\mathrm{P}_{0}=3-2 \mathrm{r}
$$

Step-3: At each $X_{K}$ position, starting at $k=0$, perform, the following test:

If $\mathrm{P}_{\mathrm{k}}<0$, the next point is $\left(\mathrm{X}_{\mathrm{K}}+1, \mathrm{Y}_{\mathrm{K}}\right)$

$$
P_{K+1}=P_{K}+4 X_{K}+6
$$

Otherwise the next point is $\left(\mathrm{X}_{\mathrm{K}}+1, \mathrm{Y}_{\mathrm{K}}-1\right)$

$$
\mathrm{P}_{\mathrm{K}+1}=\mathrm{P}_{\mathrm{K}}+4\left(\mathrm{X}_{\mathrm{K}}-\mathrm{Y}_{\mathrm{K}}\right)+10
$$

Step-4: Determine the symmetry points in other seven octant
Step-5: Move each pixel position (X,Y) into circular path

$$
\mathrm{X}=\mathrm{X}+\mathrm{X}_{\mathrm{C}} \quad \text { and } \quad \mathrm{Y}=\mathrm{Y}+\mathrm{Y}_{\mathrm{C}}
$$

Step-6: Repeat step 3 to 5 until $\mathrm{X} \geq \mathrm{Y}$

## Example-:

Given a circle radius r=10, we demonstrate the Bresenham circle drawing algorithm by determining position along the circle octant in the first quadrant from $\mathrm{X}=0$ to $\mathrm{X}=\mathrm{Y}$.

## Solution:

The initial decision parameter $\mathrm{P}_{0}$
$\mathrm{P}_{0}=3-2 \mathrm{r}=3-20=-17 \quad \therefore \mathrm{P}_{0}<0 \Longrightarrow\left(\mathrm{X}_{1}, \mathrm{Y}_{1}\right)=(1,10)$
For the circle centered on the coordinator origin, the initial point is $\left(\mathrm{X}_{0}, \mathrm{Y}_{0}\right)=(0,10)$ and initial increment terms for calculating the decision parameter are

1) $P_{1}=P_{0}+4 X_{0}+6$

$$
\begin{aligned}
&=-17+0+6 \\
&=-11 \quad \therefore \mathrm{P}_{1}<0 \Longrightarrow\left(\mathrm{X}_{2}, \mathrm{Y}_{2}\right)=(2,10)
\end{aligned}
$$

2) $P_{2}=P_{1}+4 X_{1}+6$
$=-11+4+6$

$$
=-1 \quad \therefore \mathrm{P}_{2}<0 \Longrightarrow\left(\mathrm{X}_{3}, \mathrm{Y}_{3}\right)=(3,10)
$$

3) $P_{3}=P_{2}+4 X_{2}+6$

$$
\begin{aligned}
& =-1+8+6 \\
& =13 \quad \therefore \mathrm{P}_{3}>0 \Longrightarrow\left(\mathrm{X}_{4}, \mathrm{Y}_{4}\right)=(4,9)
\end{aligned}
$$

4) $P_{4}=P_{3}+4\left(X_{3}-Y_{3}\right)+10$

$$
\begin{aligned}
& \quad=13-28+10 \\
& =-5 \quad \therefore \mathrm{P}_{4}<0 \Longrightarrow\left(\mathrm{X}_{5}, \mathrm{Y}_{5}\right)=(5,9) \\
& \\
& \quad \text { 5) } \mathrm{P}_{5}=\mathrm{P}_{4}+4 \mathrm{X}_{4}+6 \\
& = \\
& =-5+16+6 \\
& =17 \quad \therefore \mathrm{P}_{5}>0 \Longrightarrow\left(\mathrm{X}_{6}, \mathrm{Y}_{6}\right)=(6,8) \\
& \mathrm{P}_{6}=\mathrm{P}_{5}+4\left(\mathrm{X}_{5}-\mathrm{Y}_{5}\right)+10 \\
& =17-16+10 \\
& =11 \quad \therefore \mathrm{P}_{6}>0 \Longrightarrow\left(\mathrm{X}_{7}, \mathrm{Y}_{7}\right)=(7,7)
\end{aligned}
$$

| k | P | X | Y | $(\mathrm{X}, \mathrm{Y})$ |
| :--- | :--- | :--- | :--- | :--- |
| 0 | -17 | 1 | 10 | $(1,10)$ |
| 1 | -11 | 2 | 10 | $(2,10)$ |
| 2 | -1 | 3 | 10 | $(3,10)$ |
| 3 | 13 | 4 | 9 | $(4,9)$ |
| 4 | -5 | 5 | 9 | $(5,9)$ |
| 5 | 17 | 6 | 8 | $(6,8)$ |
| 6 | 11 | 7 | 7 | $(7,7)$ |




